STUDENT-LED INQUIRY

How might students be encouraged to ask and follow-up their own questions?

Introduction

At its most fundamental, inquiry-based learning is about engaging students' curiosity in the world and the ideas that surround them. As scientists and mathematicians, they observe and pose questions about situations; if their questions are too complex, they may try to simplify or model the situation; they may then try to answer their questions by collecting and analyzing data, making representations, and by making connections with what they already know. They try to interpret their findings, check that they are accurate and sensible and then share their findings with others.

This process is often missing in the school classroom. There, the teacher usually points out what must be observed, she provides the questions, demonstrates the methods to be used and checks the results. Students are merely asked to follow the instructions.

In this module, teachers will be encouraged to experience what it feels like to think like a mathematician or scientist, and reflect on the role shifts that are necessary for students to share this experience in the classroom. Teachers are shown phenomena and situations and are invited to pose and pursue their own questions. This experience is then transferred to the classroom.

Activities

Acknowledgement:
This material is adapted for PRIMAS from:
Swan, M; Pead, D (2008). *Professional development resources*. Bowland Maths Key Stage 3, Bowland Trust/ Department for Children, Schools and Families. Available online in the UK at:
http://www.bowlandmaths.org.uk
It is used here by permission of the Bowland Trust.

ACTIVITY A: ASK QUESTIONS ABOUT PHENOMENA

Time needed: 30 minutes.

In this activity, you may like to offer teachers a choice of two possible starting points:

- One involves rolling a paper cup
- The other involves a piece of computer software: *Spirolaterals*

For the first activity, you will need to provide each group of teachers with at least three different paper cups. Try to include a 'short and fat' one and a 'long and thin one' and 'one in between'
For the second, we have provided a computer microworld. Teachers will need to work in pairs using a laptop.

For the situation you choose to explore:

- Make a list of things you notice about the situation.
- What questions occur to you?
- You might begin by asking questions that start:
    - What would happen if ....?
    - What can I vary ...?
    - What effect will each variable have on ...?
- Now set yourself a problem and attempt to tackle it.

When you have experimented with the situation try to analyze your findings.

- What data have you collected?
- How have you organized your data?
- How can you explain your findings?

After teachers have explored both situations ask them to reflect on the process they have been through. Handout 2 will help them do this. Did they:

Formulate problems?

- list variables?
- simplifying and represent?

Analyse and solve?

- visualise; draw diagrams?
- systematically change variables?
- look for patterns and relationships?
- make calculations and keep records?
- make conjectures and generalisations?
- use logical, deductive reasoning?

Interpret and evaluate?

- form conclusions, arguments and generalisations
- consider appropriateness and accuracy
- relate back to the original situation

Communicate and reflect?

- communicate and discuss findings effectively
- consider alternative solutions
- consider elegance, efficiency and equivalence
- Make connections to other problems?

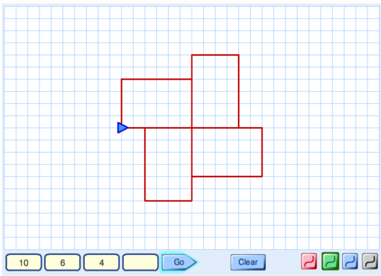
Handout 1: Phenomena to explore

Rolling cups
Look at these two paper cups.
Imagine that they are going to roll across the floor.

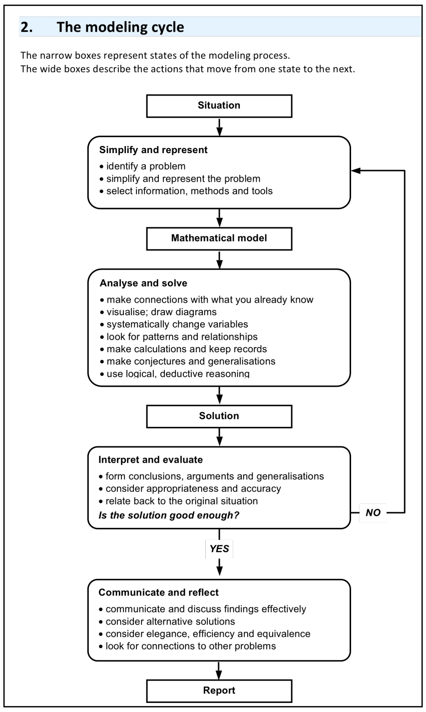
- List some possible questions that occur to you.
  "Will the cups roll in a ...?"
  "How can I predict ...?"
  "What would happen if....?"
- Make up some conjectures. These might start like this:
  " When you use this shape of cup then this is what will happen ..."
  " If you roll the cup too hard, then ..."
- Now carry out an experiment and collect some data.
  Can you *explain* and *prove* your conjectures?

Spirolaterals
Type some numbers into the *Spirolaterals* machine.
Press "Go" and watch what happens.

- How do the numbers control what is drawn on the screen?
- List some questions to explore.
  These might start like this:
  "How can we make the computer draw ....?"
  "What will happen if we .....?"
  Try and answer your own questions!
- Make up some conjectures. These might start like this:
  " When you use three numbers then ..."
  " When you repeat a number then ..."
  " If you change the order of the numbers then ..."
  Can you *explain* and *prove* your conjectures?



Handout 2: The modeling cycle



ACTIVITY B: MAKING OBSERVATIONS FROM PHOTOGRAPHS

Time needed: 20 minutes.

It is not always easy for pupils to see any connection between the real world and their lessons in school. As a result, they don't use what they learned in secondary school, even though thinking scientifically could help them understand the world better – and make better decisions.

Look at the selection of photographs on Handout 1.

- Make a list of things you notice about the situation.
- What questions occur to you?
- You might begin by asking questions that start:
  - How many ...?
  - What would happen if ....?
- Now set yourself a problem attempt to tackle it.

After teachers have explored both situations ask them to share some of the questions they have created. For example, the following selection came from one group:

Dominoes:

- Which domino is missing?
- How can you organize the dominoes systematically?
- Can you make a chain or a ring with the complete set?
- How many spots are there in a complete set? What is a quick way of counting them?
- How many dominoes are there in a complete set from (1,1) to (n,n)?

Calendar:

- How are the numbers arranged on the cubes?
- Can you draw nets and make the cubes?
- What impossible dates can be made from these cubes?

Stack of barrels

- How many barrels are in the stack?
- If you make a taller stack 4, 5, ... barrels high, how many barrels will you need? Generalize?
- How else could you stack these barrels? What other pyramids are possible?

A pavement in Germany

- Are all the paving slabs identical? What shape are they? Can you work out any angles?
- Can you draw one of the slabs accurately?
- Can you find other pentagons that tessellate?
- What other shapes can paving slabs be?

Trike with square wheels

- Does the trike run smoothly? Can you make a simple model?
- What is the height of each 'bump' on the track?
- Can you draw the shape of the 'bumpy road' accurately?
- What would happen if you had triangular wheels or hexagonal wheels?

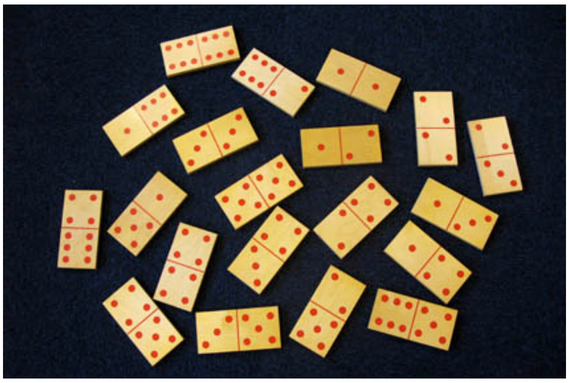
Russian dolls

- Do the tops of the heads lie on a straight line? What does this tell you?
- If you were to make some bigger dolls in this set - how big would they have to be?

Ask teachers to bring their own photographs to a follow-up session and develop questions about them. Generating questions is an activity that is essential for inquiry-based learning. We will see in a later session how students may be encouraged to develop their own questioning.

Handout 3. Photographs to explore

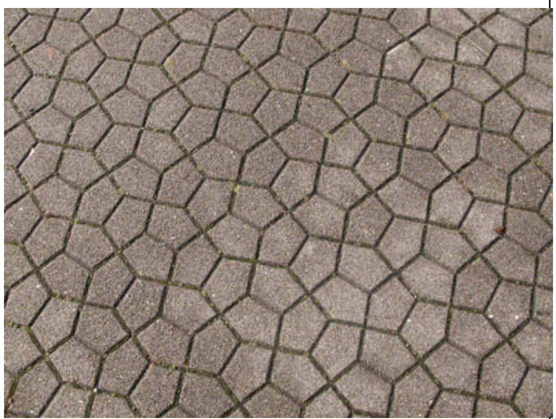
Dominoes



Calendar

Barrels

Pavement in Germany



Trike with square wheels

Russian Dolls

ACTIVITY C: OBSERVE AND ANALYSE A LESSON

Time needed: 30 minutes.

We have provided two videos of lessons for teachers to watch.
Choose just one of these.
One video uses the Spirolaterals problem from Activity A
One video uses the Building a School photographs shown opposite.
Each video lasts about 10 minutes.

As you watch each lesson, ask yourself:

- Which processes can you see in the work of these pupils?
- Can you see them:
    - Simplifying and representing the situation?
        - What questions did they formulate?
        - What simplifications and representations did they create?
        - What choices did they make of information, methods and tools?
    - Analysing and solving the model they've made?
        - Which variables did they consider?
        - What information did they collect, or guess?
        - What relationships did they formulate?
        - What calculations did they make?
    - Interpreting and evaluating the results?
        - What did they learn about the situation?
        - Were their results plausible?
    - Communicating and reflecting on the findings?
        - How did they explain their analyses?
        - What connections did they see to other problems?

Handout 4: Building a school with bottles in Honduras

1. Building a school with bottles in Honduras

Look at the pictures and:

- Make a list of things you notice.
- Write down some mathematical problems that occur to you.
- Now try to solve one problem!

First we collect old plastic bottles ..... and fill them with sand.

and make some foundations with rocks.... and start to build....

and build.... and build....

Add windows... and plaster the walls.

This building is in Honduras and is now a centre for a secondary education programme that is designed to equip and motivate young people to help their communities and to reduce poverty. The programme is particularly designed to help students develop a capacity for problem solving.

Photographs with kind permission from:
Bayán Asociación de Desarollo Socio-Económico Indígena, La Ceiba, Honduras.

Handout 5. The modeling cycle applied to the "building a school" task.

5. Building a school with bottles: the modeling cycle

Below we illustrate the modeling cycle applied to the Bottles situation.

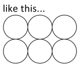
(i) Simplify and Represent
We first identify some of problems that may be asked:

- How many bottles do I need for a building like this?
- How tall is the building, and the man?
- How do the bottles fit together?
- How much sand will we need to fill the bottles?
- What about the mortar in between?
- How do the corners work?
- What about doors and windows?
- What about the roof?

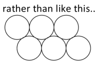
We'll focus (to begin with, at least) on a practical approach to
How many bottles do I need for a building like this?

To begin with we'll simplify the situation to assume there are 4 walls (as suggested by the angles in the bottom photograph), all the same size, and that there are no windows! We'll make calculations easier if we also assume that the number of bottles needed would not be much different if they were stacked in a 'square' fashion: i.e.

like this...



rather than like this...



We'll modify these assumptions in the second cycle of the process.

(ii) Analyse and solve
Count the number of bottles in a row.
Estimate the number of rows (you can't see them all)
Number in one wall is approximately the product of these.
Add up for 4 walls – assume the walls are the same size.

There are about 25 bottles in a row.
We can see and count only the top 7 rows clearly; these are about 1/3rd of the height
So we estimate that there are about 3 x 7 ~ 20 rows
So the wall contains about 25 x 20 ~ 500 bottles
Assuming the 4 walls are the same size gives 4 x 500 = 2000 bottles

(iii) Interpret and evaluate
This is good enough to illustrate the modeling process (and easy to report), but (and this is why it is a modeling *cycle*) if we were really serious about understanding the problem it would need to be improved by returning to tackle some of the other questions listed above.

ACTIVITY D: PLAN A LESSON, TEACH IT AND REFLECT ON THE OUTCOMES

Time needed:

- ***15 minutes discussion before the lesson***
- ***1 hour for the lesson***
- ***15 minutes after the lesson***

Now it is your turn to plan a lesson using the *Building a School* or the *Spirolaterals* situation from Activity C.
Discuss how you will:

- introduce the situation to pupils;
- introduce the idea of the modeling cycle;
- organise the classroom and the resources needed;
- answer the question "Why are we doing this in maths?";
- conclude the lesson in a way that gives pupils a better understanding of the nature of scientific processes involved.

After you have designed your lesson, compare your plan with the lesson plan supplied on Handout 6.
Discuss the differences.

It is helpful to present the lesson using a data projector. In addition, it is helpful to have a supply of the following resources available for working on the problems that arise:

- Some sample 1 litre plastic bottles
- Rulers or tape measures,
- Circular counters or coins (for working out how bottles pack together),
- Isometric dotted paper (to help with drawing and counting).
- Some copies of Handout 3 for pupils to use and discuss.

After you have taught the lesson, take some time to reflect on what happened and the processes that were in evidence.

- What questions were identified?
- Did pupils use a range of representations?
- What relationships did they find in the situation?
- What calculations did they do? Could they interpret the meaning of these?
- Were they able to communicate their conclusions effectively?
- Did your pupils feel that this was different from a normal lesson?
- Are they now beginning to appreciate how the techniques they have studied in school may be linked to unfamiliar situations?

Handout 6 A sample lesson plan

6. A sample lesson plan

The following suggestions describe one possible approach to using the photographs with students. This approach is intended to introduce them to the modeling cycle. The timings below are very tentative. This lesson outline may well stretch into two lessons in practice!

Introduce the situation, then ask students to identify problems 5 minutes

The aim of today's lesson is to see if you can use mathematics to analyse a situation.
To start with, you may not think the situation has anything at all to do with maths or science.
I want to see if you can be creative and find ways of using the things you have learned at school.

Introduce the situations carefully and vividly. Use the PowerPoint presentation on an interactive whiteboard, if possible.

These photographs were taken in Honduras. They show some people building a school out of old one-litre plastic bottles, just like the ones you buy lemonade in. They first fill them up with sand and then use them as bricks.
This is a great way of using waste materials!
What questions could we ask about this situation?

Give students two minutes to note down any problems that spring to mind, then collect their ideas on the board. For example:

How many bottles (or how much sand) will it take to build one wall?
How many bottles to build the whole building?
How do the corners work?

Ask students to identify which problems may be solved using mathematics and ask each group to choose one of these problems to work on.

Simplify and represent the problem 10 minutes

Explain that situations are sometimes too complicated to analyse as they stand. We have to simplify them before representing them with maths. Thinking with mathematics almost always involves this process.

How might we get started on the problem? Can we try a simpler problem first?
What resources could we use to help us think about the problem?
Would squared paper, isometric paper, a tape measure, a ruler help?
What kinds of diagrams might help?

Describe the resources that are available for working on the problem. Where appropriate, leave these at the side of the room, so that students can choose whether or not they use them.

SUGGESTED FURTHER READING

Learning mathematics through contextualised situations.
Boaler J. (1993) 'The Role of Contexts in the Mathematics Classroom', *For the Learning of Mathematics* 13(2)

Looking at the apprenticeship model of learning.
Brown, J. S., Collins, A. and Duguid, P. (1989) 'Situated cognition and the Culture of Learning', Educational Researcher, 18 (1), pp 32-42.

Looking at a different way to organise the Year 9 curriculum
Carter, C. (2008) 'A different way', *Mathematics Teaching,* 207, pp 38-40
http://www.atm.org.uk/mt/archive/mt207files/ATM-MT207-38-40-mo.pdf

What do pupils see as mathematical? Does it have to have numbers?
Mendick, H., Moreau, M. and Epstein D. (2007) 'Looking for mathematics' in D. Kuchemann (Ed.) Proceedings of the British Society for Research into Learning Mathematics 27 (1) pp 60 – 65
http://www.bsrlm.org.uk/IPs/ip27-1/BSRLM-IP-27-1-11.pdf

A comparison of the mathematics people use in school and out of school.
Nunes, T., Schliemann, A.D., Carraher, D.W. (1993), *Street mathematics and school mathematics,* Cambridge University Press

What is important in mathematics education?
Polya G (2002) 'The goals of mathematical education: part 1 and part 2' *Mathematics Teaching,* 181, pp 6-7 and 42-44
http://www.atm.org.uk/mt/archive/mt181files/ATM-MT181-06-07.pdf
http://www.atm.org.uk/mt/archive/mt181files/ATM-MT181-42-44-mo.pdf