TACKLING UNSTRUCTURED PROBLEMS

'Do I stand back and watch, or intervene and tell them what to do?'

Introduction

In most mathematics and science classrooms, students are provided with structured tasks and are told precisely which techniques to deploy. Students learn by following instructions. Problems and situations that arise in the world are not usually like this. Rather than being exercises in the use of a particular skill or concept, real-world problems require students to make simplifications, model situations, *choose* appropriate knowledge and processes from their 'toolkit', and test whether their solution is "good enough" for the purpose in hand.

It seems logical that if students are to learn to use their skills autonomously in their future lives, they will need some opportunities to work on less structured problems in their classrooms. This unit compares structured and unstructured versions of problems and considers the demands and challenges unstructured problems present to students and teachers.

Activities

Acknowledgement:
This material is adapted for PRIMAS from:
Swan, M; Pead, D (2008). *Professional development resources*. Bowland Maths Key Stage 3, Bowland Trust/ Department for Children, Schools and Families. Available online in the UK at: http://www.bowlandmaths.org.uk. It is used here by permission of the Bowland Trust.

ACTIVITY A: REVISING STRUCTURED PROBLEMS

Time needed: 20 minutes.

Handout 1 presents three structured problems:

- Organising a table tennis tournament
- Designing a box for 18 sweets
- Calculating Body Mass Index

These problems are of the same type as those typically found in many Mathematics and Science Classrooms. The first two are practical group tasks and the third is a computer-based task. These are, however, structured so that they lead students through the problems, guiding and making decisions for them.

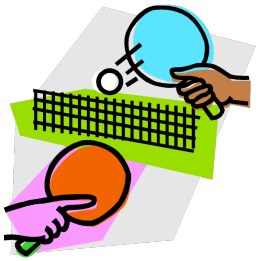
- Work through one of the structured problems carefully.
- List all the decisions that are being made for the students.
- Revise the problems so that some of these decisions are handed back to students. This will make them less structured.

For example, in *Organising a table tennis tournament*, pupils are told:

- how to code the players (A, B, C .. etc)
- to list all the matches that need to be played
- how to systematically organise these matches
- how to tabulate the order of play
- to remember that players cannot play on two tables at once.

Handout 1: Structured problems

Organising a table tennis tournament



You have the job of organising a table tennis tournament.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.

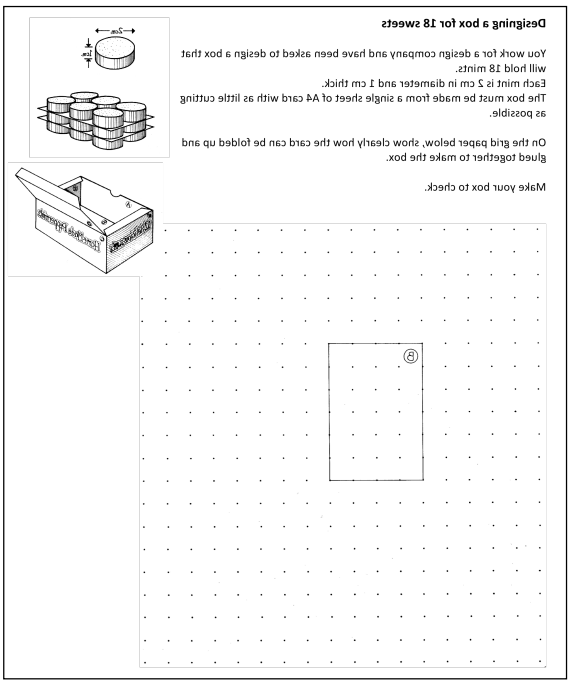
1. Call the players A, B, C, D, E, F, G
   Complete the list below to show all the matches that need to be played.

   A v B B v C ....
   A v C B v D ....
   ....

2. There are four tables at the club and each game takes half an hour.
   The first match will start at 1.00pm.

   Copy and complete the poster below to show the order of play,
   so that the tournament takes the shortest possible time.
   Remember that a player cannot be in two places at once!
   You may not need to use every row and column in the table!

Start Time	Table on which the game will be played			
	1	2	3	4
1.00	A v B			
1.30				
2.00				
2.30				
3.00				
3.30				
4.00				
4.30				



Designing a box for 18 sweets

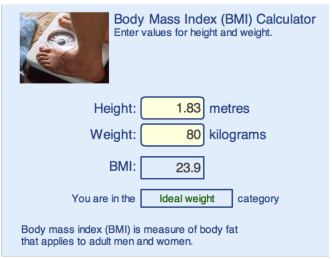
You work for a design company and have been asked to design a box that will hold 18 mints.
Each mint is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of A4 card with as little cutting as possible.

On the grid paper below, show clearly how the card can be folded up and glued together to make the box.

Make your box to check.

Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.



1. Fix the height at 2 metres - a very tall person!
   Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

   (a) What is the largest BMI for which someone is underweight?
   (b) What is the smallest BMI for which someone is overweight?
   (c) When you double the weight, what happens to the BMI?
   (d) Can you find a rule for calculating BMI from the weight?

2. Fix the weight at 80 kilograms and try varying the height.

   (a) When you double the height, what happens to the BMI?
   (b) Can you find a rule for calculating BMI from the height?
   (c) Draw a graph to show the relationship between the height and the BMI.

ACTIVITY B: COMPARE STRUCTURED AND UNSTRUCTURED PROBLEMS

Time needed: 10 minutes

Handout 2 contains unstructured versions of the same tasks that were used in activity A.

- Compare the less structured versions of the problems with the structured versions.
- What decisions have been left to the students?
- What pedagogical issues will arise when you start to use unstructured problems like this?

Some immediate issues that teachers raise are:

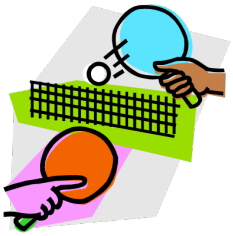
- Unstructured problems are more difficult.
- It is more difficult to plan a lesson with these problems.
- Students may not even know how to get started on them. Will we therefore need to structure them anyway?
- Students will not necessarily use what we have taught them.
- If we offer help too quickly, students will simply do what we say and not think for themselves.
- Students will generate a greater variety of approaches and solutions.
- Students may need reassurance that it is OK to try a different approach or reach a different conclusion.

Handout 3 contains some notes on the solutions to the three problems.

Handout 2: Unstructured versions of the problems

Organising a table tennis tournament

You have the job of organising a table tennis league.

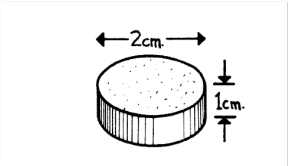


- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

Plan how to organise the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.

Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 sweets.
Each sweet is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of A4 card with as little cutting as possible.



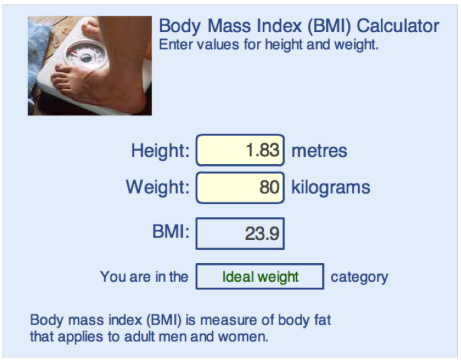
Compare two possible designs for the box and say which is best and why.

Make your box.

Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight.
What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

Investigate how the calculator works out the BMI from the height and weight.



Note for pupils: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

ACTIVITY C: CONSIDER STRATEGIES FOR OFFERING HELP

Time needed: 30 minutes.

Teachers often find it difficult to know when to give help and when to leave students struggling. If they intervene too quickly, then the students have no chance to experience what it is like to pursue an unfruitful idea, or to puzzle out a solution for themselves. If they intervene too slowly, then students become frustrated, bored and disengaged.

Handout 4 contains some practical advice when using unstructured problems. Consider this advice carefully:

- Which ideas do you normally find most difficult to implement? Why is this?
- Is there any other advice you would add to this list? Write your own ideas at the bottom.

Bruner uses the metaphor of *scaffolding* to describe the structuring that a teacher provides (D. Wood, Bruner, & Ross, 1976). The teacher encourages students to as much as they are capable of unaided and only provides the *minimum* of support to help them succeed. This support may involve reducing their choices, drawing attention to important features through questioning, or even at times demonstrating what to do. In his work with young children, Wood (1988) categorised different levels of scaffolding, from less directive to more directive: giving general verbal advice, giving specific verbal instructions, breaking the problem down, demonstrating a solution. Wood also introduced two rules of contingency:

> "Any failure by a child to succeed in an action after a given level of help should be met by an immediate increase in help or control. Success by a child then indicates that any subsequent instruction should offer less help than that which preceded the success, to allow the child to develop independence." Wood (1988)

The important idea here is that scaffolding should be removed as the student begins to cope, otherwise it reinforces dependency.

Handout 4: Practical advice for teaching problem solving

Allow pupils time to understand and engage with the problem Discourage pupils from rushing in too quickly or from asking you to help too soon.	• *Take you time, don't rush.* • *What do you know?* • *What are you trying to do?* • *What is fixed? What can be changed?* • *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for pupils by breaking it down into steps.	• *How could you get started on this problem?* • *What have you tried so far?* • *Can you try a specific example?* • *How can you be systematic here?* • *Can you think of a helpful representation?*
Encourage pupils to consider alternative methods and approaches Encourage pupils to compare their own methods.	• *Is there another way of doing this?* • *Describe your method to the rest of the group* • *Which of these two methods do you prefer and why?*
Encourage explanation Make pupils do the reasoning, and encourage them to explain to one another.	• *Can you explain your method?* • *Can you explain that again differently?* • *Can you put what Sarah just said into your own words?* • *Can you write that down?*
Model thinking and powerful methods When pupils have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	• *Now I'm going to try this problem myself, thinking aloud.* • *I might make some mistakes here - try to spot them for me.* • *This is one way of improving the solution.*
	•

ACTIVITY D: OBSERVE AND ANALYSE A LESSON

Time needed: 30 minutes.

The two video clips show students working with the unstructured versions of the same problems you have worked on. Watch Michelle using the Organising a table tennis tournament problem. As you watch the video, consider:

- How did the teacher organise the lesson? What phases did it go through?
- What resources did the teacher have available, and when were these used?
- Why were students expected to work in pairs/ small groups?
- How did the teacher introduce the problem to students?
- What different approaches were being used by students?
- How did the teacher support the students that were struggling?
- How did the teacher encourage the sharing of approaches and strategies?
- What do you think these students were learning?

Afterwards you may enjoy watching the second video clip of Judith's lesson where she asks the class to design a box to hold 18 sweets.

Michelle begins the lesson by introducing the problem and also by explaining how the students should work together. The students are then given 3-4 minutes to write their individual, initial ideas of how they will tackle the problem. This helps them to formulate their ideas and have some ideas to contribute to the group discussions. Students were then given a few minutes to discuss their ideas. The video shows students getting to grips with the problem and the constraints involved. Some for example realise that 4 tables will not be needed.

At this point, Michelle introduces the resources that are available for them to use. She emphasises that they should not rush the problem solving and that some may not get finished. She says that the important thing is to think about the approaches.

While groups work, Michelle first listens, then intervenes to promote deeper thinking:

> " You've found one approach. You've found that it is not going to quite work, is it? So what do we need to do from there?"
>
> "Re-read the problem. Look at the last two sentences."

Students use a wide range of representations and resources to solve the problem. Some used tables, while others used counters. These methods are shared in the final whole discussion.

ACTIVITY E: PLAN A LESSON, TEACH IT AND REFLECT ON THE OUTCOMES

Time needed:

- ***15 minutes discussion before the lesson***
- ***1 hour for the lesson***
- ***15 minutes after the lesson***

Choose one of the three problems that you feel would be appropriate for your class.

Discuss how you will:

- Organise the classroom and the resources needed.
- Introduce the problem to pupils.
- Explain to pupils how you want them to work together.
- Challenge/assist pupils that find the problem straightforward/ difficult.
- Help them share and learn from alternative problem-solving strategies.
- Conclude the lesson.

If you are working on this module with a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

Now you have taught the lesson, it is time to reflect on what happened.

- What range of responses did pupils have to this way of working?
  Did some appear confident? Did some need help? What sort of help? Why did they need it?
- What support and guidance did you feel obliged to give?
  Why was this? Did you give too much or too little help?
- What different strategies did pupils use?
  Share two or three different examples of pupils' work.
- What do you think pupils learned from this lesson?

If there is time, you may also like to watch the videos of the teachers as they reflect on their own lessons with the Table tennis and Sweet box problems.

FURTHER READING

The seminal text for asking pupils to think mathematically
Mason, J., Burton, L. and Stacey, K. (1982) *Thinking Mathematically,* London: Addison-Wesley

The book that inspired so much of the research into problem solving heuristics (or what to do when you are 'stuck')
Polya, G. (1957) *How to Solve It: A New Aspect of Mathematical Method*, (2nd Ed) Penguin Science.

REFERENCES

Wood, D. (1988). How Children Think and Learn. Oxford and Cambridge, MA: Blackwell.
Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. Journal of child psychology and psychiatry, 17, 89-100.