BUILDING ON WHAT STUDENTS ALREADY KNOW

How can I respond to students in ways that improve their learning?

Handouts for teachers

Contents

1 Difficulties in formative assessment

The research literature suggests that formative assessment practices are beset with problems and difficulties. These are summarised in the extensive review by Black and Wiliam (1998)[1] as follows:

Effectiveness of learning:

- Teachers' tests encourage rote and superficial learning.
- The questions and methods used are not shared between teachers, and they are not critically reviewed in relation to what they actually assess.
- There is a tendency to emphasise quantity of work and to neglect its quality in relation to learning.

Impact of assessment

- The giving of scores and the grading function are overemphasized, while the giving of useful advice and the learning function are underemphasized.
- Approaches are used in which students are compared with one another, the prime purpose of which seems to them to be competition rather than personal improvement; in consequence, assessment feedback teaches low-achieving students that they lack "ability," causing them to come to believe that they are not able to learn.

Managerial role of assessment

- Teachers' feedback to students seems to serve social and managerial functions, often at the expense of the learning function.
- Teachers are often able to predict students' results on external tests because their own tests imitate them, but at the same time teachers know too little about their learning needs.
- The collection of marks to fill in records is given higher priority than the analysis of students' work to discern learning needs; furthermore, some teachers pay no attention to the assessment records of their students' previous teachers.

[1] Black, P., & Wiliam, D. (1998). Inside the black box : raising standards through classroom assessment. London: King's College London School of Education 1998.

2 Principles for formative assessment

Formative assessment may be defined as:

"… all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet the needs."
(Black & Wiliam, 1998 para, 91)

Make the objectives of the lesson explicit
Share the objectives with students and from time to time ask students to produce evidence that they can achieve these objectives.

"Make up an example to show me that you know and understand Pythagoras' theorem."
"This lesson was about you deciding what methods to use. Show me where you did this."

Students may find it difficult to appreciate that some lessons are concerned with understanding concepts, while others are more concerned with developing inquiry-based processes.
Making objectives explicit doesn't mean writing them on the board at the beginning of the lesson, but rather referring to them explicitly while students are working. If the objectives are to develop inquiry-based processes then in plenary sessions, ask students to share and compare approaches, rather than answers.

Assess groups as well as individual students
Group activities allow many opportunities to observe, listen, and question students. They help to externalise reasoning and allow the teacher to see quickly where difficulties have arisen.

Watch and listen before intervening
Before intervening in a group discussion, wait and listen. Try to follow the line of reasoning that students are taking. When you do intervene, begin by asking them to explain something. If they are unsuccessful then ask another student to help.

Use divergent assessment methods ("Show me what you know about ...").
Convergent assessment strategies are characterised by tick lists and can-do statements. The teacher asks closed questions in order to ascertain whether or not the student knows, understands or can do a predetermined thing. This is the type of assessment most used in written tests. Divergent assessment, in contrast, involves asking open questions that allow students opportunities to describe and explain their thinking and reasoning. These questions allow students to surprise us - the outcome is not predetermined.

Give constructive, useful feedback
Research shows that responding to students' work with marks or levels is ineffective and may even obstruct learning. Quantitative feedback of this type results in students comparing marks or levels and detracts from the mathematics itself.
Instead, use qualitative oral and written comments that help students recognise what they can do, what they need to be able to do and how they might narrow the gap.

Change teaching to take account of assessment
As well as providing feedback to students, good assessment feeds forward into teaching. Be flexible and prepared to change your teaching plans in mid course as a result of what you discover.

Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.

3 Making reasoning visible

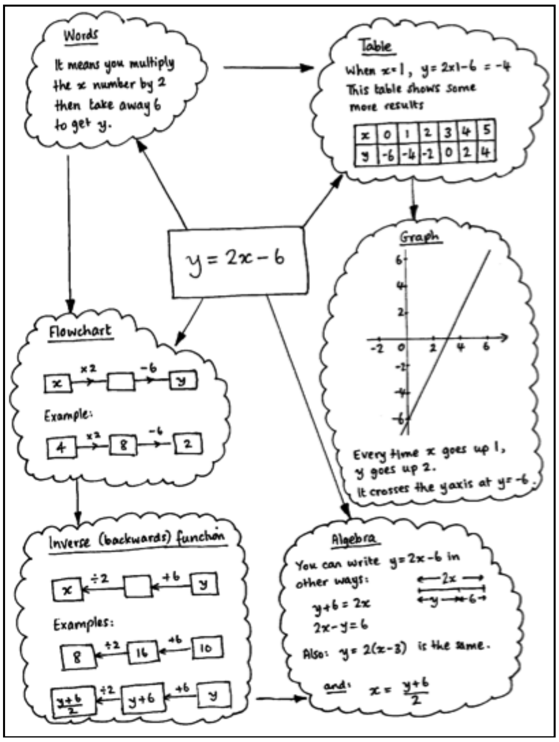
Use questioning with mini-whiteboards

One difficulty with normal classroom questioning is that some students dominate while others are too afraid to participate. In this strategy, every student presents a response simultaneously. When open questions are used, students are able to give different responses to those around them. The teacher is able to immediately assess which students understand the ideas and which are struggling.

Ask students to produce posters

Ask each small group of students to work together to produce a poster:

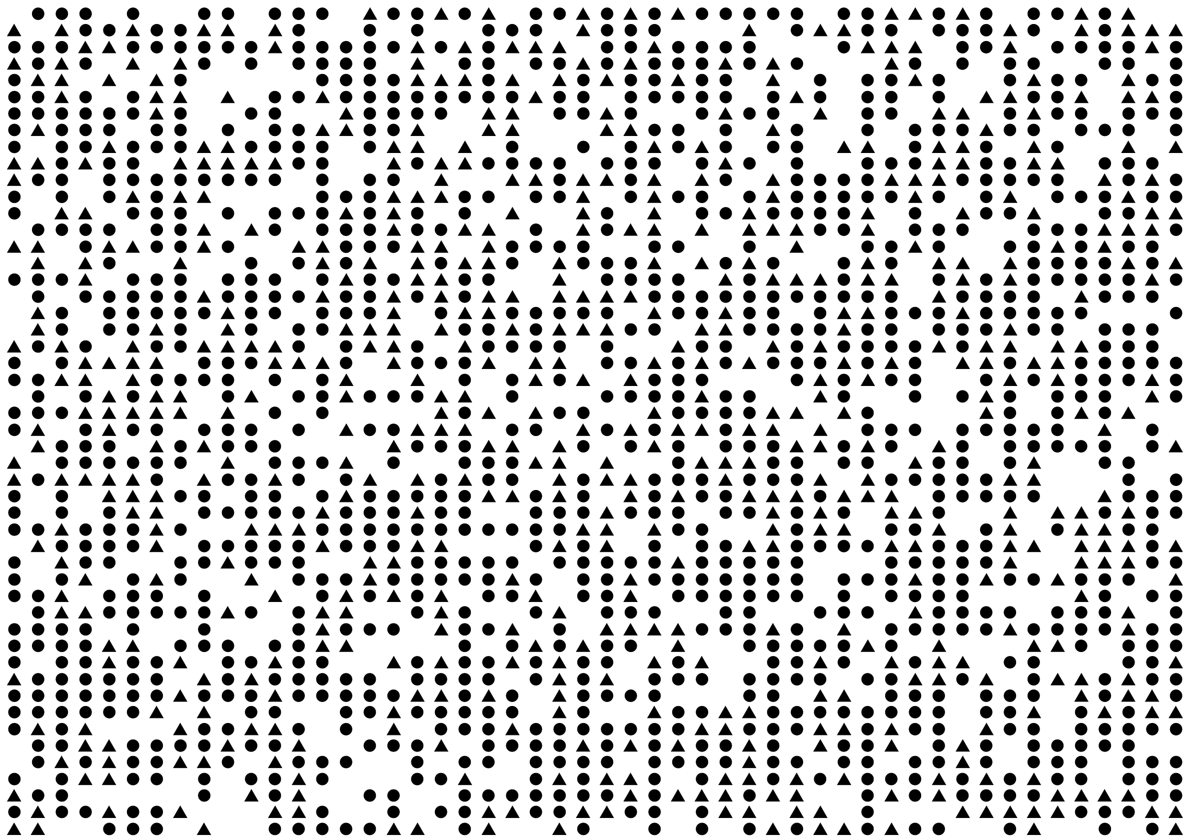
- showing their joint solution to a problem
- summarising what they know about a given topic, or
- showing two different ways to solve a given problem.
- showing the connections between a mathematical idea and other related ideas.



Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.

4 Assessment tasks and sample responses

Counting Trees



This diagram shows some trees in a plantation.

The circles ● show old trees and the triangles ▲ show young trees.
Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type?
   Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
   (a) Old trees
   (b) Young trees

Sample response: Laura

① You could multiply the number of trees in the length by the number of trees in the width and then half your answer.

② a. Old trees – 644
Young trees – 644

width – 33
length – 39.

33x39= 1287
1287÷2= 643.5 – 644

Sample response: Jenny

1. there are 38 trees in each column
there are around 11 young trees
and around 27 old ones
33 trees in each row so

11 x33 = 363
27x33= 891
1254

2.
a. 11x33 = 363 = new trees.
b. 27 x 33 = 891 = old trees.

Sample response: Woody

Sample response: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

53 old	x 25	=	1325	old
28 new	x 25	=	700	new
19 spaces	x 25	=	475	spaces
100			2500	

1325 + 1200 ÷ 2 = 1262.5
700 + 875 ÷ 2 = 787.5

Check

48 old	x 25	=	1200	old
35 new	x 25	=	875	new
17 spaes	x 25	=	425	spaes
100			2500	

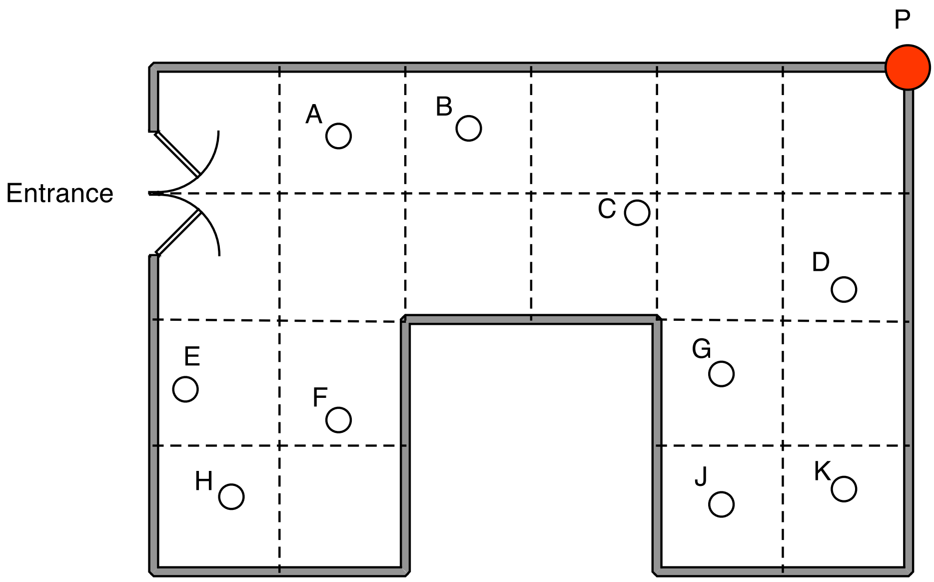
So about 1263 old trees and 788 new trees

4 Assessment tasks and sample responses (continued)

Security Camera

A shop owner wants to prevent shoplifting.
He decides to install a security camera on the ceiling of his shop.
The camera can turn right round through 360°.
The shop owner places the camera at point P, in the corner of the shop.
The plan below shows ten people are standing in the shop.

Plan view of the shop



1. Which people cannot be seen by the camera at P ?

2. The shopkeeper says that "15% of the shop is hidden from the camera"
Show clearly that he is right.

3. (a) Show the best place for the camera, so that the it can see as much of the shop as possible.

   (b) Explain how you know that this is the best place for the camera.

Sample response: Max

1. E, F and H cannot be seen by the camera.

2.

3a. The exact middle of the shop would be the place where it could see the most amount of people.

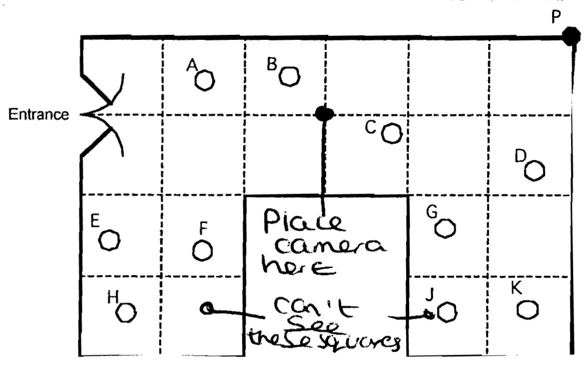
3b. Because the middle of the shop will grant the camera a larger visi of the ~~so~~ shop.

Sample response: Ellie

1. F + H

2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%. They do because if 15% of the room = 100% then to ge from 10 to 100 you divide by 10 and if you get 5 to 100 you divide by 2 and then by 10. add them together and you'll get 15%

3a+b. I think the best place for the camera is in the centre of the room because it only can't see two squares.



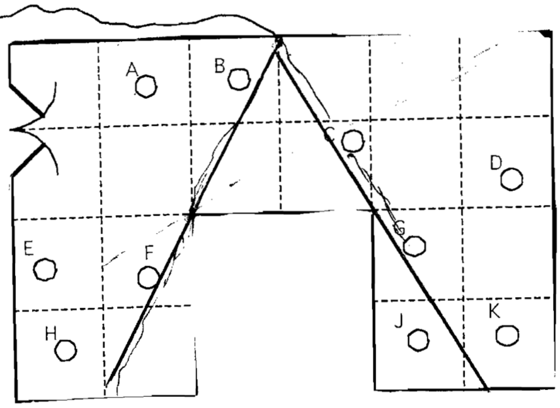
Sample response: Simon

1. F + H

2. because 3 squares a hidden from the camera 1 square is 5% so 3 squares are 15%

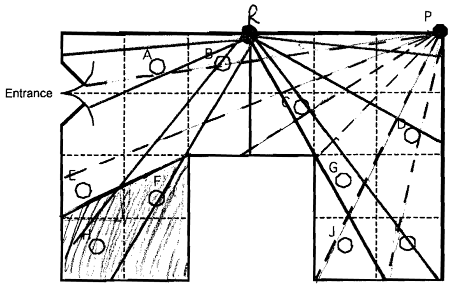
3.

a Here is the Best plase



b it can see all the carro almost everywhere

Sample response: *Rhianna*



1. He cannot see F + H.

2. There are 20 Squares. 3 squares are hidden from the camera.
Each square repersents 5%.
$3 \times 5\% = 15\%$.
This proves 15% of the shop is hidden

3.

a) ● = R
My camera

5% is hidden on one half.
5% is hidden on the other half.
This way only 10% is hidden + that space could be used for a til / trolleys.

B) I know this is the best place because it has a full veiw of all around the shop it can go ↶.

4 Assessment tasks and sample responses (continued)

Cats and kittens

Here is a poster published by an organisation that looks after stray cats.

Cats can't add but they do multiply!

In just 18 months, this female cat can have 2000 descendants.

Make sure your cat cannot have kittens.

Work out whether this number of descendants is realistic.
Here are some facts that you will need:

Length of pregnancy
About
2 months

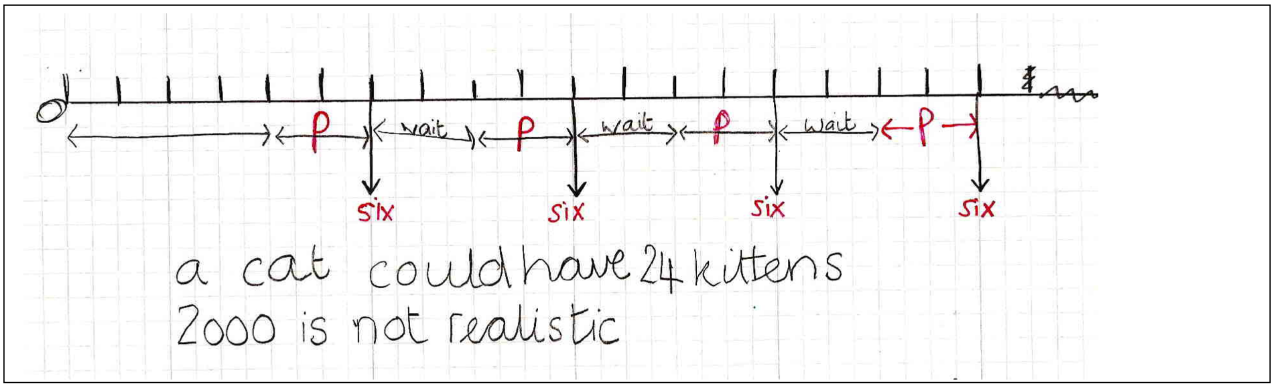
Age at which a female cat can first get pregnant
About
4 months

Number of kittens in a litter
Usually
4 to 6

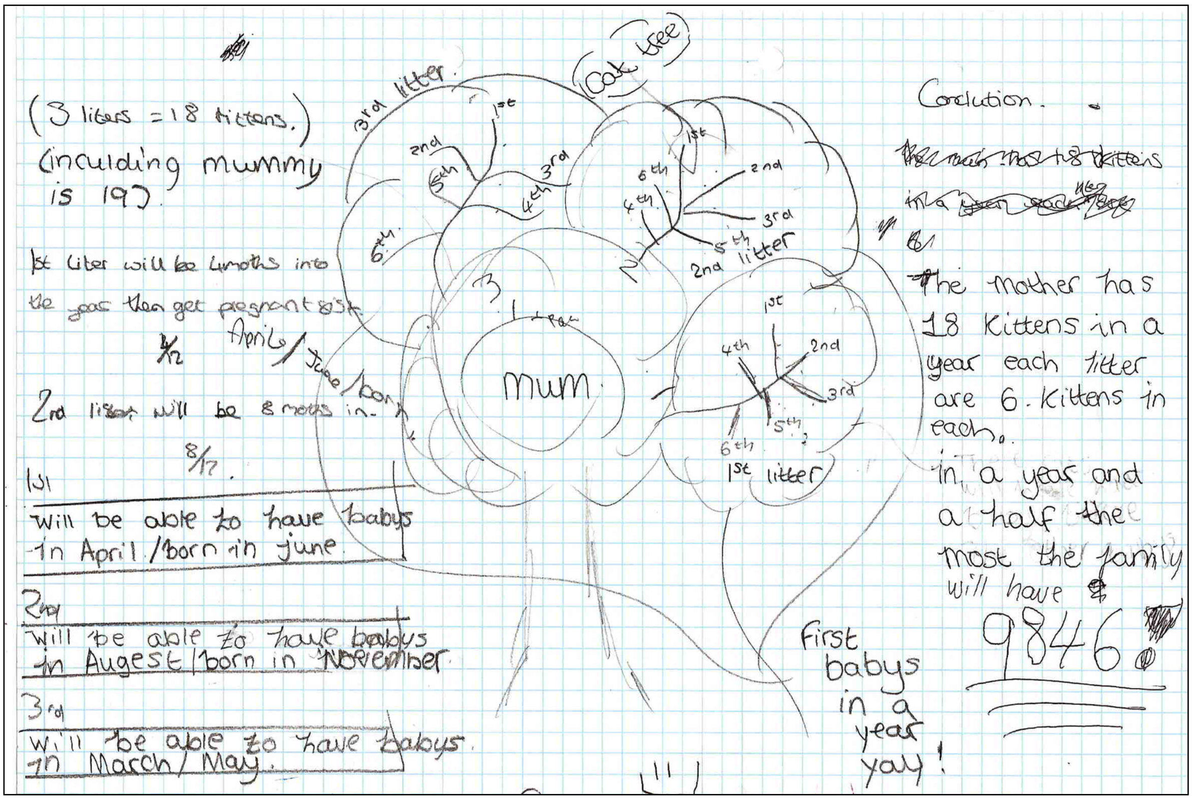
Average number of litters a female cat can have in one year
3

Age at which a female cat no longer has kittens
About
10 years

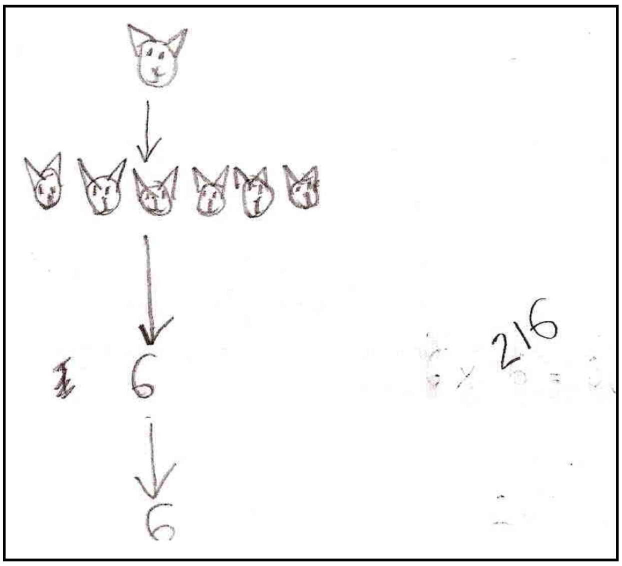
Sample response: Alice



Sample response: Ben



Sample response: Wayne



Sample response: Sally and Janet
Two students worked on this task, discussing and sharing their methods.
They used a spreadsheet.

Jan	born					
Feb						
Mar						
Apr	pregnant					
May						
Jun		6				6
Jul						
Aug						
Sep						
Oct		6	36			42
Nov						
Dec						
Jan						
Feb		6	36	216		258
Mar						
Apr						
May						
Jun		6	36	216	1296	1554
						1860

We think 2000 is a bit much in 18 months because even if each litter was 6 and nothing dies there would be 1860 though that rounds to 2000 so maybe its OK. The cat people want owners to have their cats newtured so that they use the bigger number so that people say that is a lot of cats and rush to the vets.

Primas

5 Improving students' responses through questioning

Counting Trees

Sample response: Laura

Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then halving. She does not account for gaps nor does she realise that there are an unequal number of trees of each kind.

What questions could you ask Laura that would help her improve her response?

Sample response: Jenny

Jenny realises that sampling is needed, but she multiplies the number of young trees and old trees in the left hand column by the number of trees in the bottom row. She ignores the columns with no trees in the bottom row, so her method underestimates the total number of trees. She does, however, take account of the different numbers of old and new trees.

What questions could you ask Jenny that would help her improve her response?

Sample response: Woody

Woody uses a sample of two columns and counts the number of old and young trees. He then multiplies by 25 (half of 50 columns) to find an estimate of the total number.

What questions could you ask Woody that would help him improve his response?

Sample response: Amber

Amber chooses a representative sample and carries through her work to get a reasonable answer. She correctly uses proportional reasoning. She checks her work as she goes along by counting the gaps in the trees. Her work is clear and easy to follow.

What questions could you ask Amber that would help her improve her response?

Primas

Security Camera

Sample response: Max

Max realises that F and H cannot be seen, but incorrectly thinks that E cannot be seen. He does not show any work to justify his thinking and his further statements are incorrect.
Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then halving. She does not account for gaps nor does she realise that there are an unequal number of trees of each kind.

What questions could you ask Max that would help him improve his response?

Sample response: Ellie

Ellie does not show any sightlines to justify her answers. However, she correctly states that F and H cannot be seen and that 3 squares cannot be seen. However, she may be thinking of whole squares rather than areas,. Her justification for the 15% is incomplete and poorly explained. She seems to have some understanding that 5% is one twentieth and 10% is one tenth.

What questions could you ask Ellie that would help her improve her response?

Sample response: Simon

Simon correctly states that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. However, it is possible that he thinks that 3 whole squares are hidden from the camera. He investigates the best place for the camera, and shows that the centre of a side is good but he does not investigate further. No calculations are shown.

What questions could you ask Simon that would help him improve his response?

Sample response: Rhianna

Rhianna correctly shows that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. She investigates the best place for the camera, and shows that the centre of a side is good. Rhianna clearly shows diagrams with sightlines and calculations that justify her findings.

What questions could you ask Rhianna that would help her improve her response?

Cats and Kittens

Sample response: Alice

Alice chose to represent the task using a timeline. She has only considered the number of kittens from the original cat. The computation required is accurate.

What questions could you ask Alice that would help her improve her response?

Sample response: Ben

Ben has decided to draw a 'cat tree', and tries to control for time (with some errors). The communication is reasonably clear, allowing the reader to follow the argument, but the value of 9846 is not explained and does not follow from the reasoning, since, again, only the kittens from the original cat are considered. The number of kittens per litter is made explicit.

What questions could you ask Ben that would help him improve his response?

Sample response: Wayne

Woody appears to favour a minimalist approach! He starts with what would be a time consuming pictorial representation which he then abandons in favour of a numerical representation.

What questions could you ask Wayne that would help him improve his response?

Sample response: Sally and Janet

Sally and Janet used a spreadsheet to control for both time and multiplication and their method is clear and effective.

What questions could you ask Sally and Janet that would help them improve their response?

6 Suggestions for questions

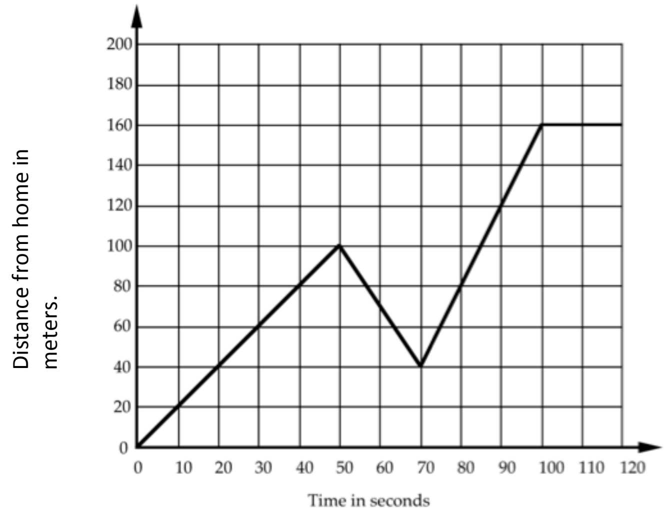
Formulate questions, choose appropriate representations and tools.	• What questions might you ask about this situation? • How can you get started on this problem? • What techniques might be useful here? • What sort of diagram might be helpful? • Can you invent a simple notation for this? • How can you simplify this problem? • What is known and what is unknown? • What assumptions might you make?
Reason logically, construct hypotheses and arguments, compute accurately	• Where have you seen something like this before? • What is fixed here, and what can you change? • What is the same and what is different here? • What would happen if I changed this.. to this...? • Is this approach going anywhere? • What will you do when you get that answer? • This is just a special case of ... what? • Can you form any hypotheses? • Can you think of any counterexamples? • What mistakes have you made? • Can you suggest a different way of doing this? • What conclusions can you make from this data? • How can you check this calculation without doing it all again? • What is a sensible way to record this?
Interpret and evaluate results obtained	• How can you best display your data? • Is it better to use this type of chart or that one? Why? • What patterns can you see in this data? • What reasons might there be for these patterns? • Can you give me a convincing argument for that statement? • Do you think that answer is reasonable? Why? • How can you be 100% sure that is true? Convince me! • What do you think of Anne's argument? Why? • Which method might be best to use here? Why?
Communicate and reflect	• What method did you use? • What other methods have you considered? • Which of your methods was the best? Why? • Which method was the quickest? • Where have you seen a problem like this before? • What methods did you use last time? Would they have worked here? • What helpful strategies have you learned for next time?

7 Assessment tasks and sample responses for concepts

Interpreting a distance v time graph

Every morning Jane walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

1. Describe what may have happened.
   You should include details like how fast she walked.



Jodie's response

Jane walked along a road for 100metres instead of walking another 30metres she took a short cut down an alleyway which took her 20 minutes She walked very quickly then She caught the bus to her college which took about 50minutes.

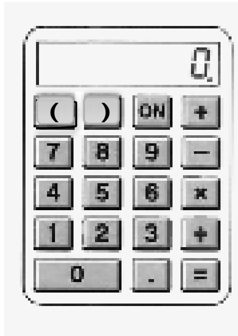
Maxine's response

when she get out she starts walking fast to the bus stop then she slows down the she picks up the speed again and then ~~the~~ her speed goes ~~con~~ constant.

Percent changes

1. Maria sees a dress in a sale. The dress is normally priced at \$56.99.
   The ticket says that there is 45% off.
   She wants to use her calculator to work out how much the dress will cost.
   It does not have a percent button.

   Which keys must she press on her calculator?
   Write down the keys in the correct order.
   (You do not have to do the calculation.)



2. In a sale, the prices in a shop were all decreased by 20%.
   After the sale they were all increased by 25%.
   What was the overall effect on the shop prices?
   Explain how you know.

George's response

① 56·99 − 0·45

② Prices went up 5%
I know this because 25% − 20% = 5%.

Jurgen's response

1. 56·99 ÷ 100 × 45 =
~~56·99 − Ans =~~
56·99 − 56·99 ÷ 100 × 45 =

2. \$56·99 = 100%
1% = 56·99 ÷ 100 = 0·5699
20% = 0·5699 × 20 = 11·398
25% = 0·5699 × 25 = 14·2475
Difference = 2·8495
\$2·85.

Enlargement

Emily's response

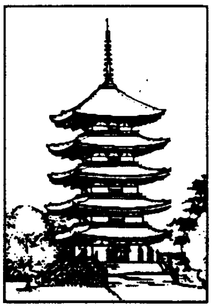
Photograph

16 cm

10 cm

Poster

?



25 cm

A photograph is enlarged to make a poster.
The photograph is 10 cm wide and 16 cm high.
The poster is 25 cm wide.
How high is the poster?

16 + 15 = 31

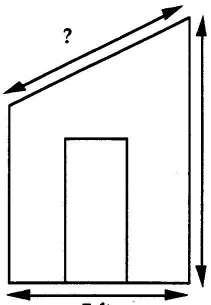
The building is 30 cm tall on the poster.
How tall is it on the photograph?

30 − 15 = 15.

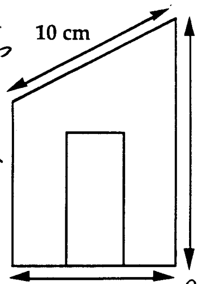
Paul's response

4. Simon is drawing a scale diagram of his garden shed.
8 centimetres on his drawing represents 5 feet in real life.

Real shed



Simon's drawing



8 = 5 . 5
4 = 2.5
= 1.25
2 = ½ = 4
5 5
2.5 2.5

5 = 2.5 = 4 cm
= 1.25 = 2 cm

(a) What is the height of the shed on Simon's drawing?

1.25 + 1.25 + 1.25 + 1.25 = 5.

(b) What is the length of the roof on the real shed?

Primas

Interpreting algebra

Britney's response

1. A cake costs c cents. A sandwich costs s cents.
I buy 3 cakes and 4 sandwiches.
What does $3c + 4s$ stand for ?

3 cakes and 4 sandwiches

2. There are ten times as many students as there are teachers in the college.
If s = the number of students in the college
t = the number of teachers in the college
Write down an equation connecting s and t.

$t = 10s$

3. There are four times as many men as there are women on a course.
If x = the number of men on the course
y = the number of women on the course
Write down an equation connecting x and y.

$y = 4x$

4. Write these expressions more simply, where you can:

a)	$a + a + a$	a^3
b)	$a \times a \times a$	a^3
c)	$a + a + b$	$a^2 + b$
d)	$a \times a \times b$	$a^2 \times b$
e)	$a + a \times b$	$a^2 \times b$
f)	$a + a + b + a + b$	$a^3 + b^2$
g)	$3a \times 4b$	$12ab$
h)	$3a + 4b$	$7ab$

If it is impossible to write the expression more simply, write NO

8 Misconceptions and errors: research findings

Learning is more effective when common misconceptions are addressed, exposed and discussed in teaching .

We have to accept that pupils will make some generalisations that are not correct and many of these misconceptions remain hidden unless the teacher makes specific efforts to uncover them.

One of the most important findings of mathematics education research has been that all pupils constantly 'invent' rules to explain the patterns that they, see around them. For example, it is well known that many pupils quite quickly acquire the 'rule' that to multiply by ten one adds a zero. Pupils then often 'over-generalise' their rules to situations that do not work. In the case of multiplication by ten, they apply it to decimals (eg 2.3 x 10 = 2.30). Similarly, pupils may decide that that multiplication always makes bigger, division smaller and then choose erroneously to multiply or divide according to their perception of whether the numbers need to get bigger or smaller.

However, overcoming these kinds of misconceptions presents the teacher with a dilemma.
When teaching multiplying whole numbers by ten, in order to present pupils with examples where adding a zero does not work, it would be necessary to stray far from the original topic, and it may, involve mathematical ideas that are, for the time being, beyond the pupils' capacity to understand.

A similar difficulty arises when teaching new procedures, where the most common approach is to apply the procedure first to simple examples and later to more complex examples. This can be counter- productive, since pupils often solve simple examples intuitively without knowing how they have solved them, and such methods cannot be used with more complex examples. So, for example, when teaching pupils about methods for solving equations it may be better to start with examples that cannot be solved by intuitive methods such as just 'spotting' the solution or 'trial and error'.

The model of simple through to more complex examples can also lay, the foundations of misconceptions. For example, teaching subtraction of tens and units and beginning with examples where no decomposition or carrying is required may reinforce the idea that you always take the smaller digit away from the larger, leading to later errors like 43 - 28 = 25.

It seems that to teach in a way that avoids pupils creating any misconceptions (sometimes called 'faultless communication') is not possible, and that we may have to accept that pupils will make some generalizations that are not correct and that many of these misconceptions will remain hidden unless the teacher makes specific efforts to uncover them. A style of teaching that constantly exposes and discusses misconceptions is needed, thus limiting the extent of misconceptions. This may be possible, as much research over the last twenty years has shown that the vast majority of pupil misconceptions are quite widely shared.

In the diagnostic Teaching Project, conducted at Nottingham University's Shell Centre for Mathematical Education, teaching packages were designed to elicit and address pupils' misconceptions during lessons. Two important features emerged. The first was that addressing misconceptions during teaching does actually improve achievement and long-term retention of mathematical skills and concepts. Drawing attention to a misconception before giving the examples was less effective than letting students fall into the 'trap' and then having the discussion.

The other major finding was that the intensity and degree of engagement with the task that pupils showed in group discussions were much more important influences on their learning than the amount of time spent on the task. Although the intensive discussions meant spending much longer on small (though important) points, there was a much higher level of long term retention overall than with classes that covered more ground superficially, but in the same time.

Askew, M; Wiliam, D. (1995) Recent Research in Mathematics Education 5-16, Office for Standards in Education, HMSO, London.

9 A formative assessment lesson plan

The following suggestions describe one possible approach to a formative assessment lesson on problem solving. Students are given a chance to tackle a problem unaided, to begin with. This gives you a chance to assess their thinking and to identify students that need help. This is followed by formative lesson in which they collaborate, reflect on their work and try to improve it.

Before the lesson 20 minutes

Before the lesson, perhaps at the end of a previous lesson, ask students to attempt one of the assessment tasks, *Counting Trees, Cats and Kittens or Security Cameras* on their own. Students may need calculators, pencils, rulers, and squared paper.

The aim is to see how able you are to tackle a problem without my help.

- *You will not be told which bits of maths to use.*
- *There are many ways to tackle the problem - you choose.*
- *There may be more than one 'right answer'.*

Don't worry if you cannot understand or do everything because I am planning to teach a lesson on this next in the next few days.

Make sure that students are familiar with the context of the problem.

Counting Trees
Does anyone know what a tree plantation is?
How is a plantation different from a natural forest?
The plantation consists of old and new trees
How might the arrangement of trees in a plantation differ from that of a natural forest?

Cats and Kittens
This is a poster made by a cats' charity, encouraging people to have their cats spayed so they can't have kittens. The activity is about what happens if you don't have your cat spayed and whether the statement on the poster is correct.
Is it realistic that one female cat would produce 2000 descendants in 18 months?
You are given some facts about cats and kittens that will help you decide.

Security Cameras
Have you ever seen a security camera in a shop or a bus? What did it look like?
Some may not look like cameras at all, but rather like small hemispheres. They may be fixed, but many swivel round. The cameras in this problem can turn right round through 360°. The drawing shows a plan view of a shop.
This means we are looking down on the shop from above.
The little circles represent people standing in the shop.

Remember to show your working so I can understand what you are doing and why.

Collect in their work and provide constructive, qualitative feedback on it. This should focus on getting students to think and reason - a Key Process agenda. Don't give grades, scores or levels! Write *only* questions below their work. Focus feedback on such issues as:

- *Representing*:
  Can you think of a different way of tackling his problem?
  What sort of diagram might be helpful?
  What assumptions have you made?
- *Reasoning*:
  How have you got this result?
  Have you checked your calculations?
  What would happen if ...?
- *Interpreting*:
  How can you test the accuracy of your estimate?
  What other sample could you have chosen?
- *Communicating*:
  I find it difficult to follow your thinking here.
  Can you present your reasoning so that someone else can follow every step?

Try to identify particular students who have struggled and who may need support. Also look out for students that have been successful. These may need an extension activity to further challenge them.

Resources needed for the lesson

You will need the following resources:

- One copy of the problem sheet per student
- Mini whiteboards
- Large sheets of paper for making posters and felt-tipped pens
- Calculators and rulers

Counting Trees

- Spare, large copies of the trees picture for groups to work on together.

Cats and Kittens

- A supply of graph paper or squared paper (if requested)

Security Camera.

- Spare copies of the plan of the shop for rough working
- Squared paper (only if requested)

Re-introduce the problem to the class 5 minutes

Begin the lesson by briefly reintroducing the problem:

> *Do you remember the problem I asked you to have a go at last time?*
> *I have had a look at your work and I have written some comments at the bottom of it.*
> *Today we are going to work together trying to improve on these initial attempts.*
> *First, on your own, carefully read through the questions I have written on your work. Use your mini-whiteboards to note down answers to these questions.*

It is helpful to ask students to write their ideas on a large sheet of paper or mini whiteboard using felt-tipped pen. This helps you monitor their work and also helps students to share their ideas later in the lesson.

Students work alone responding to your feedback 5 minutes

Allow the students some time to reflect on your comments and write their responses.

Students work in pairs to improve their solutions 10 minutes

Ask students to now work in pairs or threes. Give out a large sheet of A3 (at least) paper and a felt-tipped pen to each group.

> *Now I want you to share your work with a partner.*
> *Take it in turns to explain how you did the task and how you now think it could be improved.*
>
> *Now I want each pair to work together, comparing their ideas and the feedback I have given. Together, I want you to try to produce an answer to the problem that is better than each of you did separately.*

Go round the room, listening, assessing their thinking and making interventions asking strategic questions. Consult a copy of the progression steps for the relevant problem and decide which questions would be most appropriate for moving their thinking towards higher levels of performance. Use strategic questions like:

> *What is known and what is unknown?*
> *What are you asked to find out?*
> *How can we simplify this problem?*
> *What assumptions have you made?*

Students share their approaches with the class 15 minutes

Ask students to present their ideas and approaches to the class. Focus on their methods rather than their answers. As they respond, use the progression steps to assess their responses. In particular, focus on the quality of the reasoning and communication.

> *"We decided to count the different types of trees along each side, then multiply these numbers together."*
> *"We drew a time line along the top of the paper and then drew cats underneath to show when they gave birth."*

As students present their ideas, ask other students to comment on:

- Representing: Did they choose a good method?
- Analysing: Is the reasoning correct – are the calculations accurate?
- Interpreting: Are the conclusions sensible?
- Communication: Was the reasoning easy to understand and follow?

Students continue with the problem/ or an extension of the problem 20 minutes

Encourage students to return to the problem and continue working on it using some of the ideas that have been shared. If they have already produced a good solution, either ask them to find an alternative method, a more convincing reason, or to explore an extension.

Counting Trees
If I now showed you a very large jar of Smarties, how could you estimate the fraction that are red? Write down you method. Can you use what you learned from "Counting Trees"?

Cats and Kittens
Can you find a simpler, more elegant way of presenting your calculations to "Cats and Kittens"? Can you use a diagram of some kind?

Security Camera
There are several places that the camera might be placed that are as good as the one you have found. Try to find all the solutions. Can you convince me that these are all possible solutions? Can you explain why they all give the same coverage of the shop?

Collect examples of students' work for the follow-up discussion. Try to assess how much students have learned from the sharing session.

10 The effects of feedback on students' learning

Read the following two extracts from Black and Wiliam (1998) and respond to the questions that follow:

The dangers of giving marks, levels, rewards and rankings

"Where the classroom culture focuses on rewards, 'gold stars', grades or place-in-the-class ranking, then pupils look for the ways to obtain the best marks rather than at the needs of their learning which these marks ought to reflect. One reported consequence is that where they have any choice, pupils avoid difficult tasks. They also spend time and energy looking for clues to the 'right answer'. Many are reluctant to ask questions out of fear of failure. Pupils who encounter difficulties and poor results are led to believe that they lack ability, and this belief leads them to attribute their difficulties to a defect in themselves about which they cannot do a great deal. So they 'retire hurt', avoid investing effort in learning which could only lead to disappointment, and try to build up their self-esteem in other ways. Whilst the high-achievers can do well in such a culture, the overall result is to enhance the frequency and the extent of under-achievement."

- What are the implications of this for your practice?
- What would happen if you stopped giving marks or levels on pupils' work?
- Why are so many teachers resistant to making this change?

The advantages of giving clear, specific, content-focused feedback

"What is needed is a culture of success, backed by a belief that all can achieve. Formative assessment can be a powerful weapon here if it is communicated in the right way. Whilst it can help all pupils, it gives particularly good results with low achievers where it concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right. Pupils can accept and work with such messages, provided that they are not clouded by overtones about ability, competition and comparison with others. In summary, the message can be stated as follows:

Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils."

- What are the implications of this for your practice?
- Does this kind of feedback necessarily take much longer to give?

Black, P., & Wiliam, D. (1998). Inside the black box : raising standards through classroom assessment. London: King's College London School of Education 1998.