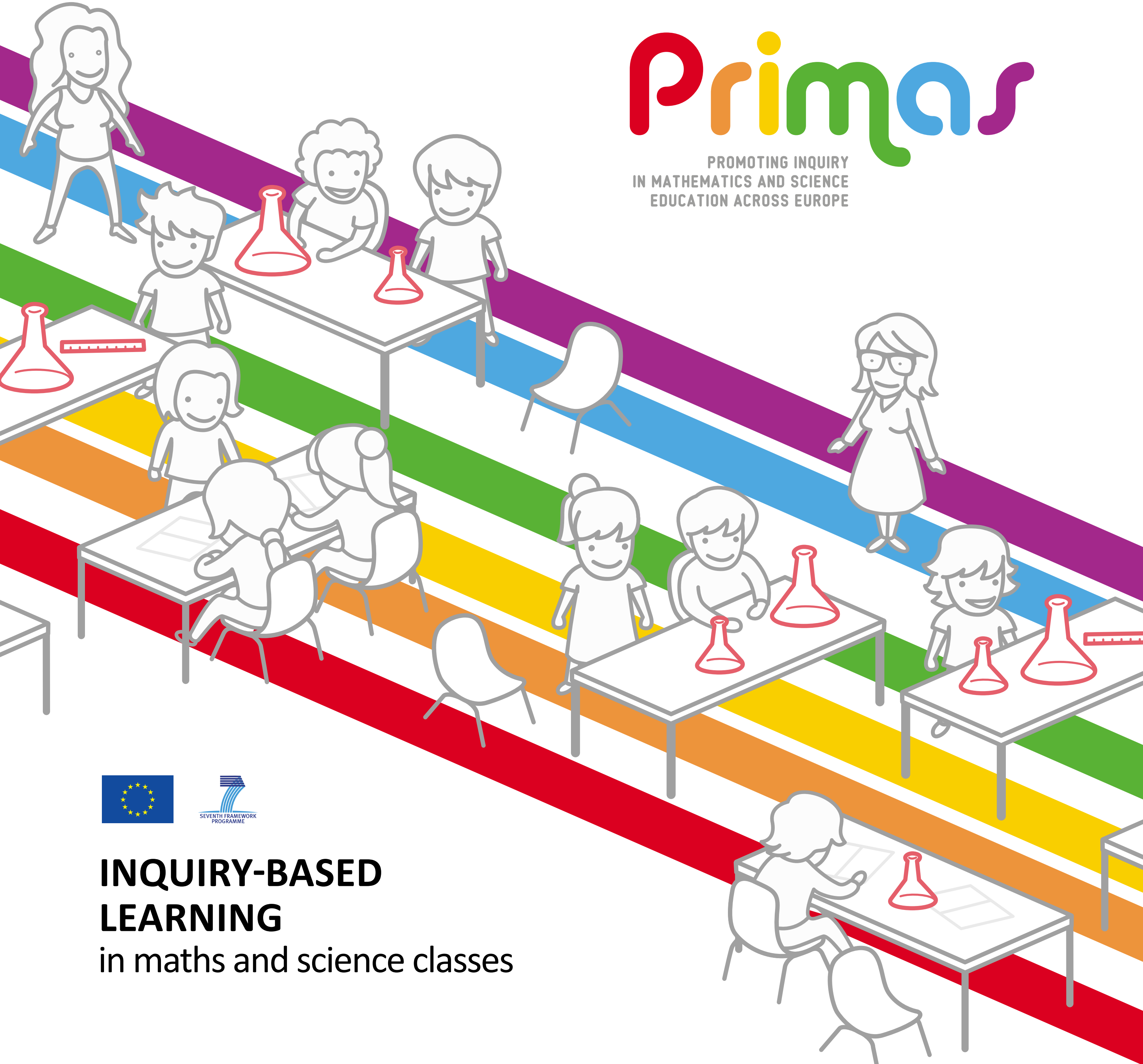
# PRIMAS

PROMOTING INQUIRY
IN MATHEMATICS AND SCIENCE
EDUCATION ACROSS EUROPE



SEVENTH FRAMEWORK PROGRAMME

# INQUIRY-BASED LEARNING
in maths and science classes

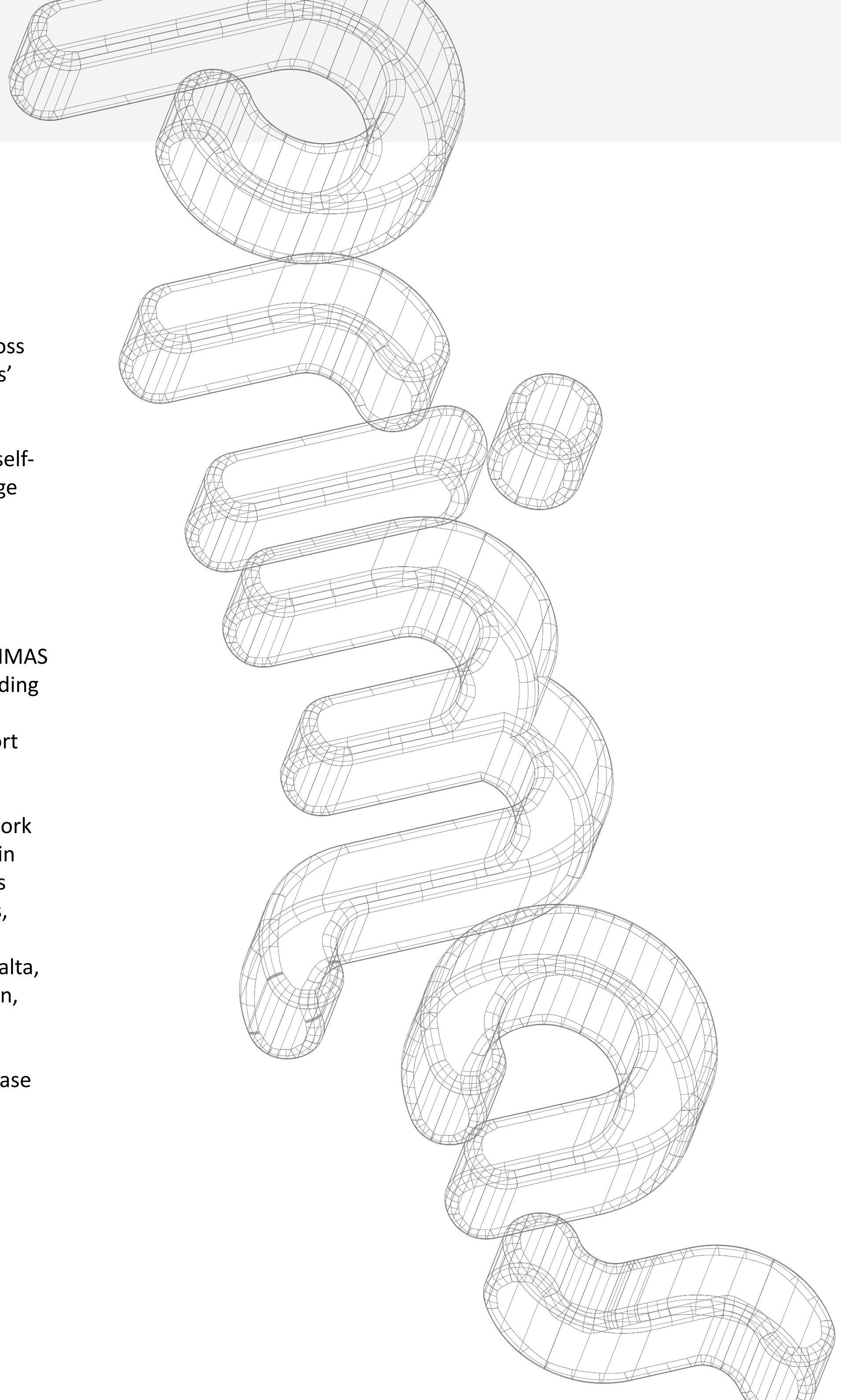
## About PRIMAS

PRIMAS stands for ‘Promoting inquiry-based learning (IBL) in mathematics and science across Europe’. IBL has the potential to raise students’ intrinsic interest in mathematics and science and it supports the attainment of important competences, such as problem-solving skills, self-directed learning and exploring new knowledge areas.

Teachers are the key players in implementing IBL pedagogies in mathematics and science classrooms and in transforming the potential benefits of IBL into real effects. This is why PRIMAS predominately aims to support them by providing teaching materials along with professional development courses and a continuous support system within ‘communities of IBL-practice’.

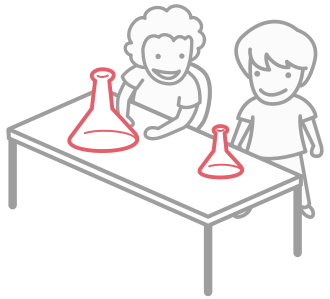
PRIMAS is funded under the EU´s 7th Framework Programme for Research, within the ‘Science in Society’ strand. The project runs for four years (2010-2013) in 12 European countries: Cyprus, Denmark, Germany (coordinating institution: University of Education Freiburg), Hungary, Malta, Netherlands, Norway, Romania, Slovakia, Spain, Switzerland and the UK.

For further information about the project, please visit the PRIMAS project website: http://www.primas-project.eu.



# Inquiry-based learning in maths and science classes

## What it is and how it works – examples – experiences



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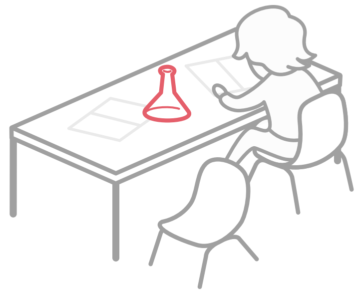
# Contents


Introduction ........................................................3
Two contrasting lessons .......................................4
What is inquiry-based learning? ...........................7
Why inquiry-based learning? ...............................9
What does it mean to teach IBL? .........................10
How can teachers learn to teach in an inquiry-based way? ...........................................11
Example tasks for inquiry-based learning ............14
Inquiry-based learning in day-to-day classes .......74
Inspiring experiences – and challenges ..............76

# Introduction

This booklet aims to provide a basic introduction to inquiry-based learning (IBL). We hope it will both inspire and provoke you to reconsider your own approaches to teaching and learning.

It is aimed at teachers and teacher educators and provides a consistent message to both audiences: We all learn best when we are actively questioning and investigating our current knowledge and practices.

In this booklet, you will be introduced to inquiry-based learning and what IBL means in practice both for classrooms and for professional development courses. At its core, it contains some short descriptions of inspiring mathematics and science lessons at the primary and secondary school levels. The detailed explanations of the tasks can be found on the PRIMAS homepage.

Towards the end of the booklet, we reflect on the benefits, practicalities and challenges involved when teachers use IBL as part of their daily, classroom practice – and hear from teachers across Europe about their own experiences.

# Two contrasting lessons

**The lessons below are, on the surface, quite similar. They are both practical experiments about the pendulum. They vividly illustrate many differences between transmission teaching approaches and IBL.**

## Mr. Shaw's lesson

Mr Shaw enters the classroom and issues copies of the textbook. 'Today', he announces, 'we are going to study the pendulum.'

He begins by showing the class a pendulum and demonstrating that the time for one swing depends on its length. He says: 'Look, if I increase the length, it swings more slowly'. He then gives each pair of students a length of string, a weight, a stopwatch and some graph paper. 'One person is to swing the pendulum, the other is to time 20 swings. One swing means going forward and back. Open your textbook and turn to page 43. Copy out and complete the table and follow the instructions carefully.'

Students notice that this page shows a diagram of the pendulum and underneath, a table with spaces for them to enter their data. The top row is labelled 'length of the pendulum in centimetres', the second row: 'time for 20 swings in seconds'. Underneath this is some blank graph paper, with axes drawn showing 'length in centimetres' labelled zero to 125 on the horizontal axis and 'time for 20 swings in seconds' labelled zero to 50 on the vertical axis. Underneath the graph are a series of instructions and questions: 'Plot a graph of your data for lengths 50 cm, 75 cm, 100 cm and 125 cm. Use your graph to find the length of a pendulum that takes exactly one second to swing back and forth.'

The students work diligently through the activity. Occasionally, a student raises their hand and ask such things as, 'Is this right? Should this line be straight?' The teacher tells students to check their work by repeating the experiment three times for each length and plotting the average of their results. Towards the end of the lesson, the teacher explains what the graph should look like and checks that students have arrived at the correct answer for the one-second pendulum. Some have, and they are congratulated.


(photo by Quadell at en.wikipedia)





## Mr. Hammond's Lesson

Mr. Hammond enters the classroom and shows students some photographs of old pendulum clocks. He asks, 'Have you ever seen clocks like these?' and, 'What do you know about them?' Then, he demonstrates two pendulums made out of string. 'Watch these carefully. Write down anything you notice and any questions that occur to you.'

First come the observations: 'One is longer'. 'The longer one swings more slowly', 'They are slowing down.'
Then, the questions start flowing: 'Can we make a clock?' 'How long does it take to swing once?' 'Can we make a timer?'
They spend some time discussing these and other questions. They try to be more specific about what the questions mean: 'What exactly does one swing mean?'

Mr. Hammond then asks the class to try to construct a pendulum with a back-and-forth swing time of exactly one second. He asks the class to collect any equipment they may need from a supply at the back of the room that includes string, drawing pens, stopwatches, rulers, pencils and 2mm graph paper.

The students begin using trial and error; adjusting the length and timing again and so on.
Mr. Hammond allows this to continue for a few minutes. Kevin claims to have completed the task, but when two other students check his work, they find it is not very accurate; it swings nearly 11 times every ten seconds. They claim that it is too short.

'Can you think of a better way, a more accurate way of making the pendulum?' Colin answers, 'If you make a long pendulum and time it to see how long it takes to swing, then if it takes, say, two seconds, you could measure the length of it and divide by two to get the length you need.' Mr. Hammond agrees that this sounds like a good idea and invites Colin to try it.

Colin makes a pendulum one metre long and times it over ten swings to find out how long one swing would take. He finds that it takes almost 20 seconds, so he says that one swing must take two seconds. He halves the length of his pendulum and tries again. To his surprise he finds that it takes about 15 seconds, so one swing takes 1.5 seconds. He is puzzled, 'This doesn't seem logical.'

'So what do we do now?' asked Mr. Hammond. 'How can we find out how the length affects the swing time?'

There is a long pause. Perhaps prompted by the availability of the graph paper, one student suggests using a graph of swing time against length. Others agree. The students then get to work. They decide for themselves what scales to use.

'We'll need to get some more results because it won't be a straight line,' says Susan. 'How do you know?' asks another. 'Because the 50 cm one would have swung once every second, but it was 1.5 seconds', answers Susan.

The students construct further pendulums of different lengths and draw graphs. They produce curves and from this they interpolate to find that for a one-second pendulum, the length of the string should be about 25 cm. They check this by making the pendulum and are delighted to find that it works.

The teacher concludes the lesson by asking students to present their reasoning to the whole class.

*'For me using IBL was a very positive learning experience. This is because when students are given the chance to explore, they come out with ideas that might not have occurred to me.'**

(Luca Gallea, teacher from Malta)

* Real names of teachers are withheld, but well-known to the editorial team.

Photo: Daniel Aguirre





# What is inquiry-based learning?

**Some of the differences between the two lessons are described in the table below. Which lesson do you think is more powerful?**

| Mr Shaw's lesson | Mr Hammond's lesson |
|---|---|
| The teacher poses the questions that are to be explored. | The teacher introduces a stimulus to the class and invites students to observe, describe and pose questions. He awakens curiosity. |
| The teacher gives each pair of students the equipment they will need. | The students are allowed to select the equipment they need. |
| There is no room for predicting and testing. Possible mistakes and misconceptions are avoided. | Predictions are discussed and tested. For example, students assume that the relationship between length of pendulum and time is linear and test this. |
| The task is completely structured by the textbook. Students make very few decisions. They mainly follow instructions. | Students are allowed to tackle the problem in any way they wish. For example, they are allowed to use trial and error. They make decisions for themselves. |
| The teacher tells students to check their work for accuracy. | The students check each others' work for accuracy. |
| The teacher mainly instructs and gives information and evaluates work. | The teacher challenges, questions and provokes students to think for themselves. Students present and evaluate each others' work. |

Mr Shaw's lesson illustrates many of the features of traditional approaches to teaching and learning. The teacher is the instructor and knowledge provider. The role of the students is passive; they are expected to follow instructions, learn factual knowledge and practice procedures through routine exercises.

In contrast, Mr Hammond's lesson stresses the importance of students' active participation; they are posing questions, making decisions, designing experiments, predicting, exploring alternative methods, discussing, collaborating, checking each others' work, summarising and communicating results. The teacher does not stand back and take a mere 'facilitating' role, but rather joins in, questioning and provoking students to reason and explain their thinking.

Inquiry-based learning is not about using new tasks. It is not about using practical experiments. It is of course true that the tasks and materials must offer students the opportunity to make decisions for themselves, but the tasks and materials do not in themselves guarantee inquiry-based learning. It is rather a perspective on learning that creates a new learning culture in the classroom.

The diagram below is an attempt to summarise the learning outcomes, the classroom culture, the learning environment and the roles of teachers and students in the inquiry-based classroom:

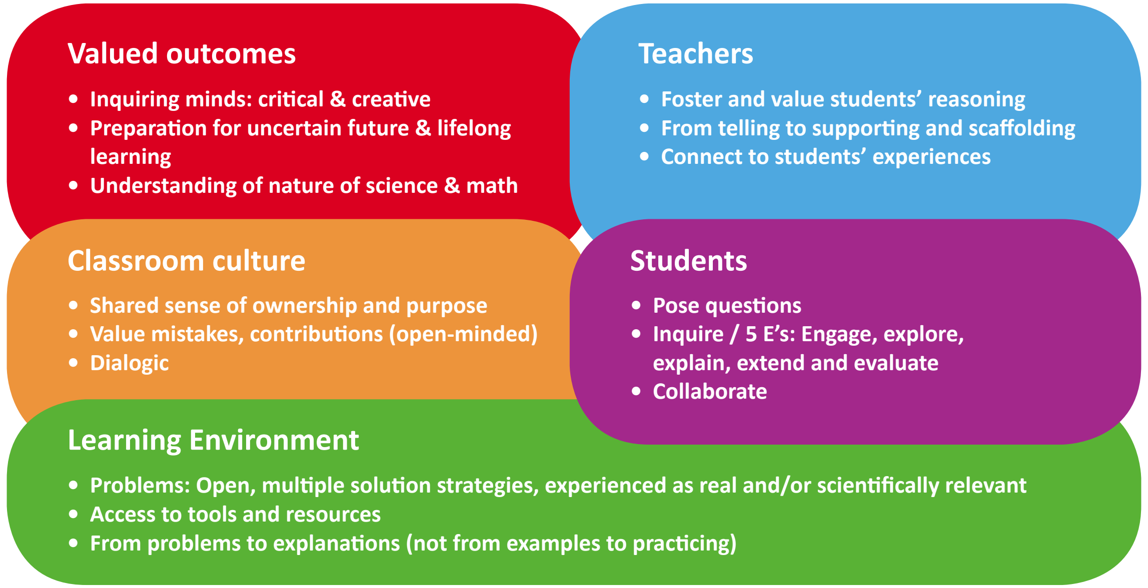



Figure 1. Multifaceted understanding of IBL



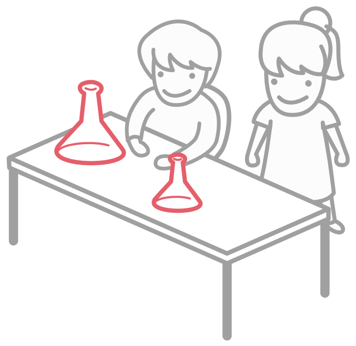
What is inquiry-based learning?

This understanding of IBL opens the door to numerous possibilities for us to implement IBL in our classrooms. These range from working with a short, simple task that perhaps makes use of only one aspect of inquiry to tackling a bigger, more complicated problem that involves students in the entire cycle of inquiry (i.e. exploring situations, planning investigations, experimenting systematically, interpreting, evaluating and communicating results).

Teaching in an inquiry-based way means that we are confident that our students can learn actively; that they can learn to provide interesting answers to problems; that they can learn to explore situations, communicate their results and give reasons for their ways of proceeding.
Inquiry-based learning does not mean that we neglect mathematical or scientific content.
In fact, we argue that students develop competencies that support and deepen their understanding of the content.

*'I remember the very first lesson, there were quite a few of them saying, "Well, what do I do?" And they don't do that anymore. I gave them the 'Golden Rectangles' problem, I didn't talk about it I just gave them the sheet and told them it was one of these problem-solving things and asked them to look at it and they were just like "OK. Let's have a look."'*

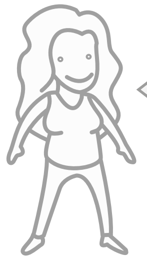
(Euan Saunders, teacher from the UK)



# Why inquiry-based learning?

Changing from transmission teaching to inquiry-based learning means that students and teachers have to get used to new ways of learning and teaching – which can be considered as a challenge at the beginning. So, why bother?

In our modern, fast developing society, it is no longer sufficient to prepare students for their futures by having them learn facts. It is hard to predict the knowledge that will be needed in the years to come. We do know, however, that tomorrow's employers will still expect their employees to be able to solve non-routine problems, to analyse data, to discuss with colleagues, to communicate their results and to work autonomously. Where should students acquire these competences if not in school? Inquiry-based learning is a means to support students in developing these competences.



*'In my understanding, the essence of the method [IBL] is that certain tasks and problems can be discussed without external help or with the least possible external help. When providing freedom for thoughts, children may learn to cooperate, to pay attention to each others' thoughts.'*

(Zsófia Borbás, teacher from Hungary)





# What does it mean to teach IBL?

## Choosing appropriate tasks

In order to develop students' abilities to perform such processes, teachers need to use different kinds of tasks than those commonly presented in textbooks in which the recall of basic knowledge and practising of routine procedures dominate. The teacher has to propose sufficiently rich situations, with authentic questions that have to be accessible to students, but yet not too easy. You will find inspiring examples in this booklet – ranging from primary to secondary level and from mathematics to science.

## Using different pedagogies

Pedagogies must make a shift away from a 'transmission' orientation in which teacher explanations, illustrative examples and exercises dominate, towards a more collaborative orientation in which students work together on interconnected and challenging tasks. Here, the teacher's role is different and includes: making constructive use of students' prior knowledge; challenging students through effective, probing questions; managing small group and whole class discussions; encouraging the discussion of alternative viewpoints; and helping students to make connections between their ideas.

## Establishing a supportive classroom atmosphere

The classroom atmosphere is considered to be a key feature in the efficient implementation of IBL. It is important to establish a culture where there is not a knowledgeable authority but, instead, ideas are respected and accepted according to their foundation and how they are supported by evidence and logical thinking. In this atmosphere, mistakes are considered to be learning opportunities and there is a shared sense of ownership and purpose.

*'My contact with IBL has helped me to change my way of teaching from the traditional teacher centred to a student centred way. I now realise that students have more fun when they are involved in active roles. It is vital that students are the centre of the teaching and learning process. Before, I saw myself as the expert and the students as recipients. My role was to fill-up the recipients with knowledge. I used to look upon knowledge as absolute, unquestionable. Now I see things differently. Students are given the chance to question everything, to discuss in groups how to go about solving problems.'*

(Gabriel Abela, teacher from Malta)

How can teachers learn to teach in an inquiry-based way?

# How can teachers learn to teach in an inquiry-based way?

Moving towards an IBL approach raises many pedagogical issues, such as:

- How can I encourage my students to ask and pursue their own questions?
- How can I help students to follow up these questions in profitable ways?
- What if students do not make progress – how do I intervene without 'taking over'?
- How can I teach students to work cooperatively and to learn from each other?
- What about conceptual knowledge – surely students cannot be expected to reinvent mathematical or scientific concepts for themselves?

The project PRIMAS provides materials for teachers' professional development. They can be used either by teacher educators in courses or by groups of teachers working on their own. They can be found on the PRIMAS homepage: www.primas-project.eu.

The professional development materials are structured in a way that systematically tackles such questions. There are seven units, each of which is designed to occupy about three hours. In the first hour, teachers consider their own practice, watch videos of contrasting practices, discuss the specific pedagogical issues and plan lessons. The second hour is spent trying out different pedagogical practices in the classroom. In the third hour, teachers share experiences, reflecting on their learning and consider how their findings relate to the research. The units are outlined in the table below:

**1. Student-led Inquiry.**
In this unit, you are presented with phenomena and are then invited to pose and pursue your own questions. You thus experience what it feels like to think like a mathematician or scientist and are invited to try a similar activity with your own students and reflect on the outcomes. For example, what questions occur to you about this photograph?

Photo cc-by-sa Thesupermat, Wikimedia Commons

**2. Tackling Unstructured Problems.**
This unit invites you to consider the decisions that you make for students when you present them with structured problems. It invites comparison of structured and unstructured versions of problems and considers the demands and challenges unstructured problems present in the classroom. You are invited to try out unstructured versions of textbook problems in your own classroom and report back on your experiences.

**3. Learning Concepts through IBL.**
This unit considers how the processes of inquiry-based learning may be integrated into the teaching of mathematics and science content. Often, these two aspects of learning are kept separate: We teach content as a collection of facts and skills to be imitated and mastered, and/or we teach process skills through investigations that do not incorporate important content knowledge. The integration of content and process raises many pedagogical challenges. The processes under consideration here are: observing and visualising; classifying and creating definitions; making representations and translating between them; finding connections and relationships; estimating; measuring; quantifying; evaluating; experimenting; and controlling variables.

**4. Asking Questions that Promote Reasoning.**
This unit contains a selection of stimuli designed to help you reflect on: characteristics of their questioning that encourage students to reflect, think and reason; ways in which you might encourage students to provide extended, thoughtful answers - without their being afraid of making mistakes; and the value of showing students what reasoning means by 'thinking aloud' as problems are worked on in collaborative classroom environments. As before, you systematically try to develop your own classroom questioning and report back to your colleagues about what happens.

**5. Students Working Collaboratively.**
This unit is designed to offer you the opportunity to reflect on the characteristics of student-student discussion that benefit learning: to recognise and face their own concerns about introducing collaborative discussion; to explore research techniques for promoting effective student-student discussion; and to consider your own role in managing student-student discussion. You plan and carry out discussion-based lessons and report back to your colleagues about what happens.

**6. Building on What Students Already Know.**
This unit considers the different ways you can use formative assessment techniques to make effective use of students' prior knowledge. It focuses on the following questions: How can problems be used to assess performance? How can this assessment be used to promote learning? What kinds of feedback are most helpful for students and which are unhelpful? How can students become more engaged in the assessment process?

**7. Self and Peer Assessment.**
This unit encourages discussion of the following issues: How can we help students to become more aware of IBL processes, and their importance in problem solving? How can we encourage students to take more responsibility for their own learning of IBL processes? How can students be encouraged to assess and improve each other's work?

# Example tasks for inquiry-based learning

In the following, you will find several examples of tasks. There are tasks for the primary and secondary levels, and for mathematics and science lessons. Teachers in various European countries have tested them all, and you can see a complete description of each task and its relevant worksheets on the PRIMAS website: http://www.primas-project.eu.

Each of these IBL tasks is subject-related and based in curriculum content. However, equally important to the tasks' design was their potential use in helping students achieve competency in such areas as estimating, argumentation and giving presentations.

The collection of tasks has also been designed to provide ideas and impulses, and hopefully motivation for teachers to develop and implement their own IBL questions and tasks for use in their classrooms. Allow yourself to be inspired – and you will be surprised by your students' creativity, progress and development.

*'These tasks really do work! They make possible a completely different kind of teaching and learning – and are lots of fun!'*

(Marcel Winter, teacher from Germany)

Photo: Patrick Bronner





## Example Tasks

To float or not to float? That is the question (S 4-9) ...........16
Animal footprint (S 4-9 or 10-14) ........................................20
Crossing paper strips (M 4-9 or 10-14 ) ..............................23
The giant chair (M 6-9 or 10-14) .........................................27
Farm animals (M/S 6-9) .......................................................30
Water supply (M/S 6-9 or 10-14) .........................................33
A mathematical morning (M 10-14)......................................37
More coffee or more milk? (M 10-14 or 15-18) ..................40
Conditions of germination (S 10-14) ...................................44
Iron swings (S 10-14 ) ..........................................................47
The candle experiment (S 10-14 or 15-18) .........................50
Define it! (M/S 10-14 or 15-18 ) ..........................................53
Algebra pyramids (M 12-14 ) ...............................................55
Beads and formulas (M 15-18) ............................................58
(Cr)easy! Paper strip folding (M 15-18) ..............................61
Playing with tiles and colours (M 15-18 ) ...........................64
Keep it cold! (S 15-18) ........................................................67
Shake torch (S 15-18) ..........................................................71

**M = Mathematics**

**S = Science**

The numbers in parenthesis correspond to the appropriate age groups for the given task.

# To float or not to float? That is the question

'The stone sinks 'cos it hasn't learnt how to swim yet.' 'Can you make a potato float?' If your students or your children wonder whether an object would float/sink in water, why some objects float/sink, or how to make an object that sinks or floats, then this activity might interest you. Its aim is to let young students explore and discover what factors contribute to the objects' behaviour when immersed in water. It is designed for students to learn how to utilise and learn from the processes of inquiry. At the same time, the activity lays the foundations for developing an understanding of concepts such as forces, buoyancy, the Archimedes' principle and mass density. These concepts are often perceived as difficult to grasp, even for older children.

**Class time needed: one or two lessons**

## Pedagogical issues

Students are to make hypotheses about whether particular objects sink or float, and explain why. Then they are to investigate what happens when they put the objects into water. Different parts of the investigation are: observation of phenomena; classification of which objects float/sink; making conjectures about why they sink/float; interpreting results; finding patterns and relationships; documenting the findings (orally or in written form); and devising the next step for further systematic inquiry.

We need to supervise students in conducting the investigation without being the one who plans everything. Also, we need to be aware that the students need time to discuss/test their results against their hypotheses, and to reflect: Was my guess correct? Why, and why not? According to the students' age and maturity, they might come up with different types of hypotheses, which are often based on the object's weight, size, shape or type of material. Care needs to be taken in order for students to feel appreciated; regardless of whether their answer is correct, or not.





To float or not to float? That is the question (S 4-9)

Flexibility is required in this case, as the scope of the investigation will vary according to the children's age(s), maturity and pre-knowledge. This is discussed in detail in the **Professional Development Module 2: Tackling unstructured problems**. In the next section, we give examples of classroom practice where the task was carried out with five-year-old children. The task itself can be extended for further study of subsequent topics at higher grades.

## Classroom practice

The task was given to a group of five-year-old children by pre-service teachers (Sortland, 2012). Before the children put objects in the water, they stated their hypotheses. One child thought, 'It's just a wooden block, so it will float.' But another argued, 'No, it will sink, 'cos it's so heavy!' Notice that the first child based the hypothesis on the type of material (wood), while the second child based it on the estimation of weight ('it's heavy'). It was noted that whenever a child guessed correctly, then the others who guessed wrongly started to contemplate/rethink instead of feeling embarrassed. There was thus an atmosphere of children being appreciated 'despite' their guesses, and this is important in order to create a classroom environment that is conducive for inquiry. Even quotes that seemed very unlikely were not dismissed: 'A person floats, and princesses can't sink, can they?' (They were about to put a princess doll in water) or, 'Look! She's drowning! Maybe she should have got a life vest?'

During the activity, the children were allowed to decide what to do next after receiving hints (in the form of subtle questions) from the teacher. Some children started to attach some nails to a Styrofoam ball. Sortland reported that they became curious when presented with the question, 'What do you think will happen to the ball? Will it stay floating when we attach many nails to it?' (Sortland, 2012). The questions motivated students to take the lead and do more systematic experiments to discover how many nails were needed in order to get the ball to sink.

Children's explanations of the phenomena were related to their pre-knowledge of the subject, such as, 'Those that float lay on top', and, 'those that sink go to the bottom'. Those children knew that the meaning of the words float and sink were related to the position of the object in the water. Another child explained, 'The Styrofoam ball floats because it's strong enough'. His explanation referred to his swimming lessons.

Concepts, such as weight and type of material, arose as the children worked with the task. In the following example, we show how concepts such as volume, forces and buoyancy came up from the children's own explanations, without necessarily using the terms explicitly. 'The water helps the wooden block to stay up, it uses the invisible forces it has.' One of the pre-service teachers wrote, 'I asked the children whether they knew why the water level rose when we pushed the balloon down. Jon just said that, 'the water rose more and more'. 'Maybe I know!' replied Kristin,

'it's 'cos the balloon pushes the water away!'... 'When a balloon is being pushed in the water', wrote the teacher, 'they learnt about the upward force from the water to the balloon (buoyancy)'. The other concept that came up during the activity was density. In connection with this concept, the children used the following expressions:

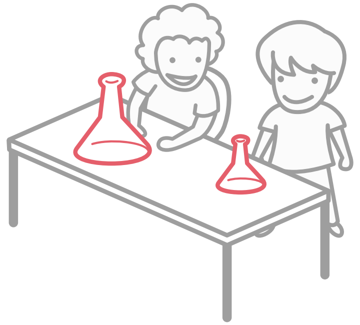
- A lot of air/little air inside the object, 'A piece of wood floats because it has a lot of air in it'.
- The object is lighter/heavier than water, 'The princess doll is heavier than water'.





To float or not to float? That is the question (S 4-9)

What the children have learned may be concluded from this reflection made by a pre-service teacher about the children's thinking: 'The children experienced the joy of knowing that they were right. They also had the chance to see the connection between heavy and light objects and their behaviour in water. They also saw that the weight alone was not the only reason that made an object float or sink, but also how the objects were shaped. Some big objects could be a bit heavy, but somehow float. In the cases where the children provided wrong hypotheses, they still had the pleasure of formulating their explanation with their own, new words about what made objects float or sink'. (Sortland, 2012).



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**References:**
Leif Wedøe in L. Wedøe (2005). Fysikkaktiviteter i barnehage og småskole. Cappelen Forlag, Oslo, 45-54.
Quotes in the classroom practice are taken from a lecture by Merete Ø. Sortland: 'Naturfagomgrep i barnehagen – NatGrep', presented in National Seminar for Science Teacher Educators in Hell, Norway 16.11.2012.

**Photos:**
Maria I.M. Febri (Various Objects), Wibeche Anita Døvik (Would the Potato Float?), Nina Haug Saltnes (Pushing Balloon)

**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1581/Floating+or+sinking/view.do?lang=en



Science · Ages: 4-9 years

# Animal footprint

This activity promotes the interpretation of observations and creativity. It builds on the knowledge of ecology, then preferably moves on to food chains. The activity does not have only one solution, but instead, promotes various solutions. Here, student ideas and proposals can be a good starting place for them to argue their point of view.

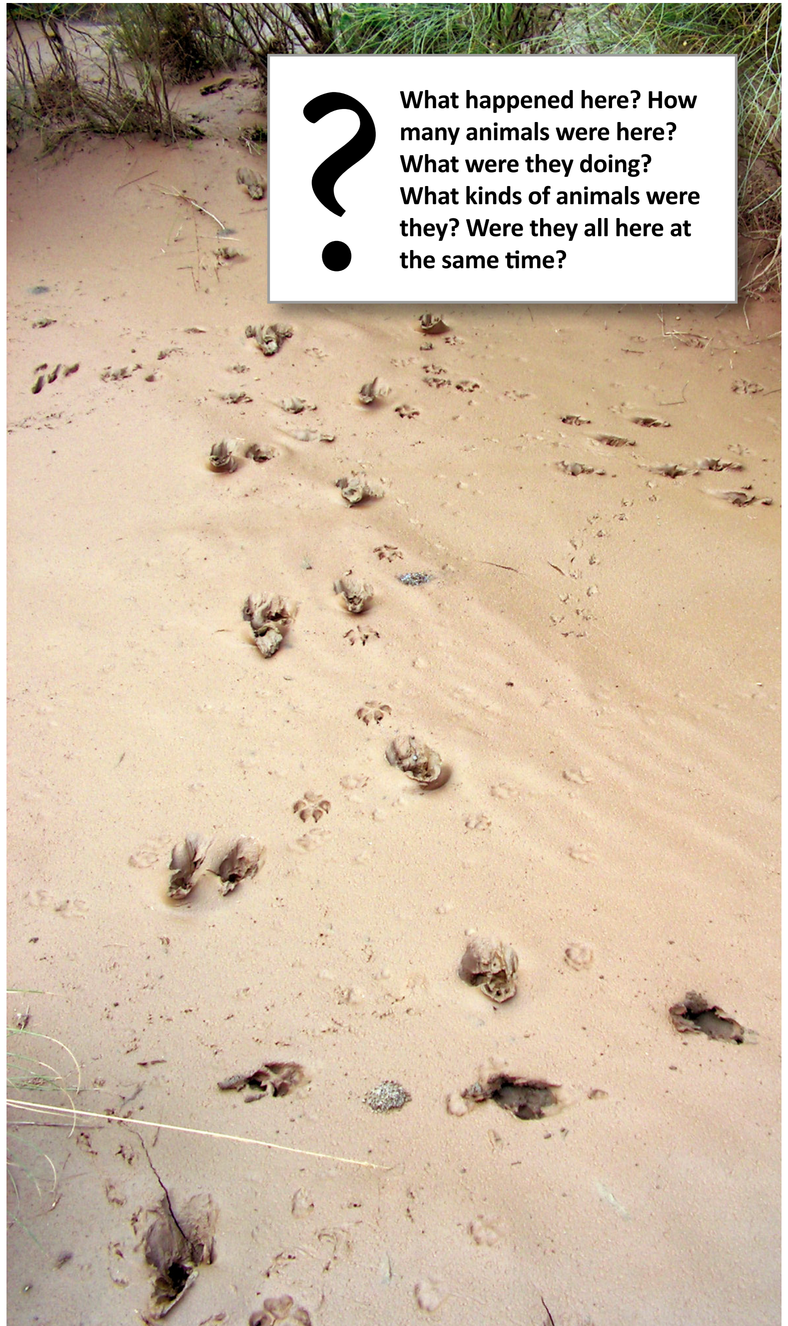
**Class time needed: one lesson**

## Pedagogical issues

The aim of this task is to promote students in interpreting observations and arguing their viewpoints. Students are supposed to make suggestions about 'the story' behind the footprints. The task supports IBL because it focuses on the inquiry aspects of interpreting, evaluating and communicating results. At the same time, the activity lays the foundations for developing an understanding of ecological relationships, species knowledge, footprints, food chains and competition. These concepts are often interesting, even for older children.
According to the students' age and maturity, they might come up with different types of interpretations. We will need to allow time for students to explore the task and to argue their proposals.










As teachers, we have a particularly supportive role in this activity, stimulating the students to think for themselves, to formulate their own questions, and to search for answers to their questions. We have to promote students to think and reason out loud, give them examples if they get stuck and ask critical questions. Our intention here is to help take thinking to a higher level.

## Classroom practice

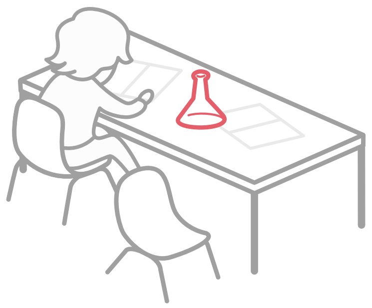
Pre-service teachers in Norway gave this task to a group of 12-year-old children. Students came up with many good suggestions about what could have happened. It was a challenge, though, to get students to understand that there is not only one correct answer. After the lesson, the pre-service teachers were supposed to comment on their experiences with this kind of task and the exploratory approach. Here are some of their responses:

*'I think students find these tasks fun, and they had to use their knowledge in a different way than they were used to. So I think that they were very motivated by this way of working.'*

*'It is important to be well prepared for the many inquisitive questions to be answered. I also learned that this is a way that captivates students. Children find the task amusing and when doing it, they gain new knowledge and it contributes to increased interest in science.'*

*'I have learned that it is popular among students and not so difficult to implement. You also get good conversations with students and it's fun to listen to their thoughts and explanations.'*

*'I also learned that it is wise to let children be active participants in class and try to use their words and explanations of why things happen. When we can create some curiosity and wonder among the students, we get a lot more out of teaching.'*



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Inspired by http://www.pbs.org/wgbh/nova/education/activities/2117_ash_01.html
**Photo:**
Anja Strasek (Desierto de Tabernas near Almeria, Andalusia, Spain)
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1320/Animal+footprint/view.do?lang=en







# Crossing paper strips

In this situation, students will be supported in their inquiry of shapes they can make when they cross two paper strips and examine the intersection between both pieces of paper.

From their experimentation, it is expected that students investigate and describe different geometrical shapes and identify specific characteristics, such as the parallelism of their sides, their lengths, properties of diagonals and (relations between) the internal angles.

**Class time needed: ca. 2 lessons**

## Pedagogical issues

This activity is intended for primary school or, depending on the complexity of the relevant curriculum content, for lower secondary school. It will help your students to identify properties of different types of quadrilaterals and to establish connections between them. (See **Professional Development Module 3: Learning concepts through IBL**.)

In a completely open version, you could allow students to cut their own paper strips and ask them to experiment and pose questions, formulate hypotheses and try to draw conclusions. Some questions that might come to the students' minds are: What happens if I cross two rectangular strips of equal (or of different) widths? In which ways can I produce a kite? Will there always be quadrilaterals, or can I get a triangle (or a pentagon)? (See **Professional Development Module 1: Student-led inquiry**.)

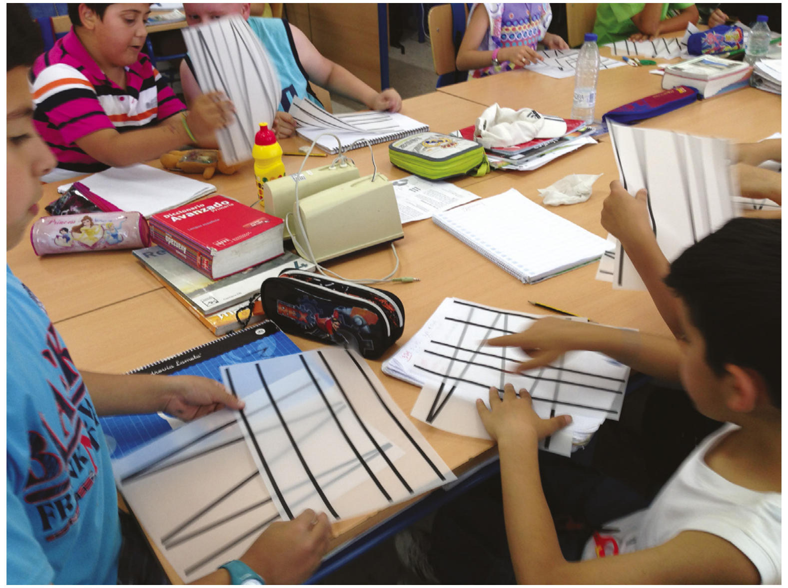
In a more structured version, or after a discussion of initial findings, you could provide students with some prepared paper strips and even a few questions. During the activity, it is important that your students' reasoning is focused on the main variables of the situation: (1) the properties of the bands (parallel edges or not) and (2) their crossing angle. Basically, the fact that the longer sides of the paper strips are parallel or not, connected with the crossing angle, gives rise to different kinds of quadrilaterals. In a more advanced level, it is also possible to get triangles and pentagons, depending on the way the band strips are crossed.

At the end, your students should be encouraged to connect the properties of the paper strips and the crossing angle with the kind of shape formed in their intersection, getting to possible definitions of parallelograms, trapeziums and trapezoids. You might extend the activity in different ways. For instance:

1. Lost paper strips: Ask students to make a shape crossing two paper strips and to cut it with scissors. Ask students to exchange their shapes. The problem is: your classmate made this shape with two paper strips but they lost them. Can you cut the strips your classmate has just used?
2. Surface: Crossing two paper strips, could you find the one with the biggest/smallest surface?
3. Hidden shapes: some shapes have been made by crossing paper strips, but a part is hidden. Could you help us find out what the shape is?

## Classroom practice

This task was implemented in a year four class in Atarfe (Granada, Spain). According to the teacher, his students had already learnt about polygons, quadrilaterals and parallelograms. Therefore, the situation did not seem to be very challenging for them.



Nevertheless, he thought it was worth giving it a try. The teacher prepared the material for his students: onionskin paper containing different lines and some scissors. Students had to cut out their paper strips, getting strips with longer sides parallel, but with different widths, and also paper strips with the longer sides not being parallel. As soon as they cut out the strips, students started playing and noticing things.





Crossing paper strips (M 4-9 or 10-14 )

In order to be sure that students understood the situation, the teacher started asking students to cross two strips with parallel edges and the same width. Quickly, students noticed that they could form squares. Some of them also noticed that rhombuses could be formed. The teacher asked the children to work in groups and think of and list the properties of the situation that led to getting a square. After several discussions, they came up with three properties, which were written on the blackboard: (1) strips with parallel edges, (2) strips of the same width, and (3) perpendicular crossing.

Now that students knew how the situation worked and the kind of results they had to look for, the inquiry phase started:

> **Can you find other kinds of shapes? What are the conditions for getting a particular type of shape? Write down in your notebook the shapes you get, together with the conditions.**

Students worked in groups of three to four, and the teacher circulated in the classroom offering support and asking effective questions. Soon, problems appeared. Students got rectangles easily and most of them were able to list the conditions, but were confused when looking for rhombus, rhomboids, trapeziums and trapezoids. Many of the students tried to find answers by looking at the parallelogram classification on the wall, but this stereotyped version of each parallelogram was not very helpful.

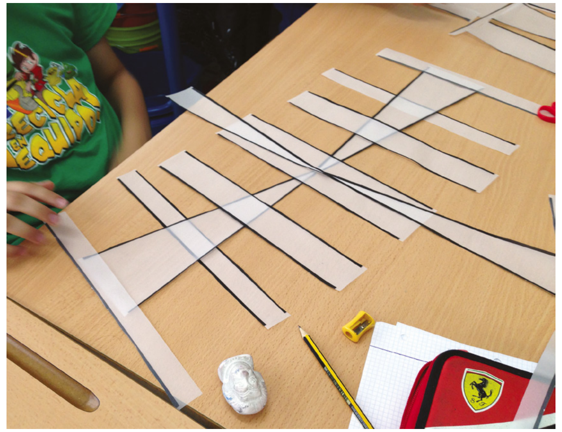


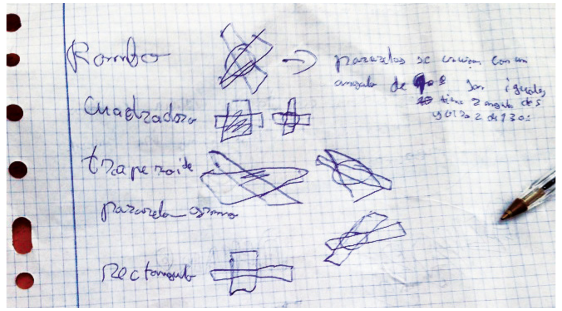
Mathematics · Ages: 4-9 or 10-14

After the inquiry phase, the teacher asked students to share their findings. Students had to explain the shape they had found and the conditions needed to get it, which gave rise to a very interesting debate about parallelograms, quadrilaterals and their properties.

In the next session, students worked in groups on a big poster. The aim was to structure and formalise their findings.

After the implementation of the task, we realised that students' initial knowledge of certain geometrical shapes was very limited. Through this inquiry process, they were able to develop a deeper understanding of shapes and their respective properties, look for and find connections between these and come up with new definitions.





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**Photos:**
Fco. Javier García
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1601/Crossing+paper+strips/view.do?lang=en







# The giant chair

Huge, over-sized chairs are sometimes used for advertising in front of furniture stores. Such an object can be used for interesting maths classes. After an introduction, the lesson can start with questions posed by the students themselves, for example: 'How tall is the giant who uses this chair?' 'How many normal-sized chairs can be placed on the seat?' The various issues are handled by the students in groups and corrected in peer feedback.

**Class time needed: ca. 2 lessons.**


## Pedagogical issues

This is a classic modelling task. Showing a photo of a person standing next to the giant chair demonstrates size comparison and is a great way to introduce the topic. In the first phase, mathematical questions are collected using the 'think/pair/share' method. The students work in small groups of three or four to find solutions to the questions. They can work with the photo to come up with a scale that in turn, helps then calculate the proportions. The other information students need to answer the questions can be taken from their common, everyday knowledge or researched on the internet. To conclude the task, students make posters that show their results and calculation methods. The posters are displayed, and during a gallery walk, students assess their own work and also receive feedback from their classmates and teacher. An alternative presentation method is to have students record their results on handouts and use these to create a booklet as described below.

The giant chair (M 6-9 or 10-14)

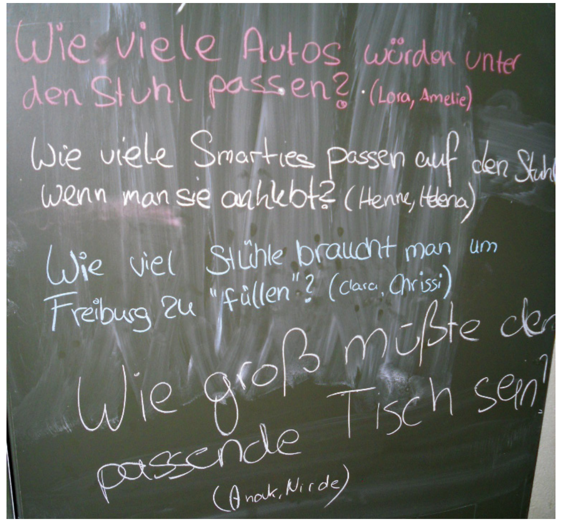
## Classroom practice

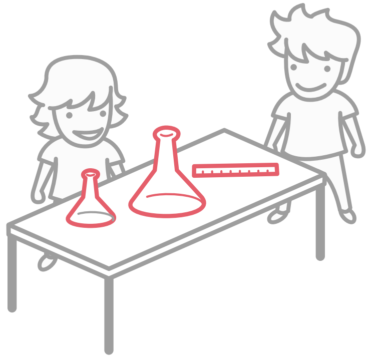
This first-hand account excerpted from a school's yearbook shows how one class experienced this task:

*In our first lesson after the school holidays, our maths teacher told us about a new furniture store in Freiburg (Germany). He was especially excited about a giant red chair at the store's car park (typical maths teacher!) So that's why he immediately got someone to take his picture in front of the chair and showed it to us at school. Our tall teacher seemed so tiny in front of this huge chair! But we used this photo to help us think of various, related mathematical questions and collected them on the blackboard.*

*Then we formed small groups, and each one chose one of the questions. That's when it got really hard, because then we had to answer our question! We used the photo with our teacher for a scale and found the rest of the information we needed in the internet.*

*To determine our answer, we made detailed calculations – and gave logical reasons for the calculations we chose. We also discussed a lot with other groups about our way of solving the problem. Finally, we wrote down our solution method and the answer to our question on a solution handout. It was all really a lot of work! But when we were finished, all the groups' work was collected and we made a kind of booklet called 'Mathematics and the Giant Chair'. We sent it to the furniture store, and its manager promptly invited our class to lunch!* (Class 8a, Friedrich-Gymnasium Freiburg, Germany)



**Author and photos:**
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**References:**
Katja Maaß, Primas Workbook: You need maths in life. Heurekas word problems for children ages 8-10, Freiburg 2011, p. 13; Friedrich-Gymnasium Freiburg, Jahresbericht 2012/2013.
**Link to the PRIMAS website:**
http://www.primas-project.eu/artikel/en/1645/Maths+around+the+giant+red+chair/view.do?lang=en







Mathematics · Ages: 6-9 or 10-14

# Farm animals

Many of the things we eat and drink come from farm animals: sausages, ham, eggs, butter, milk, etc. Which food and beverage products come from what animals? How are these products included in our diets?

**Class time needed: 2 lessons (with a week's break)**

## Pedagogical issues

Most students who live in cities have limited opportunities to have first-hand contact with farm animals. Instead, 'city kids' normally only see photos of farm animals in books or on TV. Occasionally, students may have the chance to visit a zoo or farm where they can see live animals. However, farm animal products are used everyday.

Within this task, we give students the opportunity to obtain experience with data collection, systematic observation, and communicating the results of their observations. They need to work collaboratively in groups, respect the opinions of others, and reach a conclusion as a team. They will learn where food elements come from, how to represent collected data, and how to read tables and bar charts. These are usually the very first data representations for students, particularly in this age group.

## Lesson plan

The first lesson starts with classifying different foods according to the animal(s) that provides them. We bring different products (the lesson may be linked to a common breakfast in class) and/or give students pictures of eggs, milk, other dairy products (cheese, yoghurt and butter), and meats (ham, salami etc.) and ask them to prepare a poster with each animal and the products it provides. They can discuss and use books and the internet to find the necessary information. Students present their posters at the end of the lesson. Preparing the posters usually brings a more systematic view of the content. The homework assignment is to carefully watch and record which kinds of food from animals they







eat during the following week. Students will find their own ways of recording. Some of them may write down a list of products in chronological order and thus provide a 'raw data table'. Others may prepare a table with a separate line for each animal and make a cross in the respective row for each product consumed from this animal. Some may use colours or icons for recording.

The second lesson starts with a presentation of the students' records and an evaluation of the different recording methods. The next task is to collect the information of the whole class and to merge it into a common representation. We may introduce a bar chart and fix specific colours for each animal (i.e. chicken – green, cattle – blue, pig – orange). Students are asked to colour the appropriate number of cells (one cell for each product they had consumed) in the appropriate colours on a prepared picture. Afterwards, they cut out and pin or glue their cells on the classroom bar chart which then represents the consummation of the whole class. In this preparation method, students do not need to know how to add up numbers greater than ten, as during seven days of data collection, students usually consume each product no more than seven times.

After completing the bar chart, students can examine it to find out information such as: Which product was consumed the least (or most) or which animal contributed the most to diet, and so on. Other bar charts (e.g. only for meat or for all products together), can be derived.

## Classroom practice

This task was implemented in a year 1 primary school class. For the students, it was their first encounter with preparing a poster, working on a project and participating in on-going team work. Students showed great interest in the problematic. The ability to read the first conducted bar chart came quite naturally and students grasped the concept very well.

The teacher was interested in observing students during their first project work in groups. Each child had a particular role within their project team. However, they did not always manage to keep to their role – most likely due to their age and lack of experience. Students were also not always able to deal with the task properly right from the start, but in the end they found the common conclusion. Importantly, all of them obtained some (first) experience in collecting data and representing it with the help of tables and bar charts.

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**Photos:**
Dana Strejčková (Classroom), Ernst Vikne (Cow portrait)
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1680/Farm+animals/view.do?lang=en







# Water supply

This task focuses on water shortage, a severe problem in a number of European countries. This same problem is common in much of the world and half of the planet's population is expected to face an insufficient water supply by 2025.

In the activity, students have opportunities to investigate concepts involved in mathematical measurement and simple statistics. Students also have opportunities to integrate their scientific reasoning on water and environmental awareness in solving the problem presented in the activity.

**Class time needed: Sequence of lessons**

## Pedagogical issues

The activity is designed for four, forty-minute sessions. The first session deals with background information on the water shortage problem and readiness questions to newspaper articles on the subject (cf. PRIMAS website http://www.primas-project.eu/artikel/en/1683/Water+supply/view.do?lang=en.) If needed, background information and questions can be adapted to better fit students' reading proficiency. Further, if students are familiar with technology, an introduction to Google Earth can take place. The tool can be used in the next sessions to facilitate students' work. The final three sessions are for modelling and solving the problem and for documenting the results. The activity is designed for group work in groups of three to four. Additionally, students are asked to document their results by writing an individual letter. This will help us in identifying how each student perceived the problem and its constraints, as well as the solution to the problem.

## Water import

Last week, people all over Cyprus received a water conservation advisory via mail. In that advisory, the head of the Cyprus Water Board talked about the island's reliance on desalinisation plants saying, 'We don't desalinate lightly, without being aware of the consequences. It is energy-consuming … and this causes (greenhouse gas) emissions for which Cyprus has to pay fines'. Cypriot officials recently decided to sign a contract with a nearby country to import more than 12 million cubic meters over the summer, starting at the end of June.

**What do you think: from which country is Cyprus probably importing water? What would you suggest, and why?**

Students may begin work on the problem in groups of three or four. Before students begin the decision making in this project, we can discuss qualitative and quantitative information, the importance of each, and have a full class discussion about how qualitative measures (quality of port facilities) may be used. We can also discuss how different variables (e.g., distance between countries, water price) can be used together, how they can be sorted, or even discuss in a qualitative manner concepts like average and range (e.g., average in each variable).

Students can then plan their investigation into finding the best place that can supply Cyprus with water. After solving the problem, students can write a letter that explains their solution. They can also prepare (in groups) a poster in which they can better explain and document their results. During student explorations, we can move through student groups and encourage them to think of 'more dimensions' of the problem. The activity can be implemented in secondary school and even in primary school. The complexity of questions raised and of approaches to solutions will vary according to the students' age and ability.

The following describes an implementation in a primary school. Usually, students at this age (6-9) tend to think of 'unidimensional' solutions (e.g., best country is the one that sells the cheapest water or the country that is closest). We need to encourage them to think of all available data. The activity required students to document their results in written form. If this is difficult, each student can explain orally how they worked.





## Classroom practice

In an implementation of the activity in a year 3 classroom of nine-year- olds, students created a number of interesting models. They engaged with the problem and were able to appreciate its significance.

They enjoyed working with computers, and especially with Google Earth. A large number of student groups used software to explore nearby countries and their landscapes. They added placemarks and used the 'ruler' to calculate the distances. Discussion on the meaning of kilometre and its relation to other units of measurement took place between the teacher and students in many groups. In a number of groups, students discussed tanker oil price and water price. Students, however, failed to successfully use the provided data, and they based their final choice (Lebanon) partly on the provided data and on their calculations, but without providing a coherent solution.

Of course, some of their approaches were not successful. Two groups, for example, just made random calculations, only partially using the provided data. One group, for example reported that importing water from Greece was the best solution, since the water price was cheap.

In contrast, in two groups students ranked the countries for each variable (distance, oil price) and based their final solution in combining these two variables in a rather intuitive way. Their solution was importing water from Lebanon, on the grounds that Lebanon is closest to Cyprus (it was the first country in respect to the variable 'distance') and oil cost is not very expensive (however, it was also not the cheapest in respect to the variable 'oil cost') – a quite sophisticated solution.

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**Photos:**
Nicholas G. Mousoulides
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1683/Water+supply/view.do?lang=en







# A mathematical morning

'What a nice morning are we having today! What a lot of mathematics you can find everywhere!' Greeting a class in this way is meant to be a starting point for inquiry learning in mathematics. The students themselves define the questions which they want to work on in order to mathematically describe their daily, morning activities.

'How many litres of water (tea, juice…) do I, or does our class, or does the whole school drink every morning for breakfast?' 'How much tooth paste am I going to consume during my life-time?' Many more questions may come to the students' minds. The various questions are to be collected in class, reviewed, improved and then worked on in small groups. The results are to be presented on A3 posters.

**Class time needed: The task has been used for one 45 minute lesson as well as for a sequence of between four and six lessons.**

## Pedagogical issues

Probably the most difficult and most interesting phase in the inquiry cycle is searching for everyday problems that can be solved and discussed in the classroom. Students should be encouraged to find questions and problems they had not even thought to be of mathematical nature. Support

for this phase of work can be found in the PRIMAS Professional Development Module 1: Student-led inquiry.

Since this task is mainly about problem posing, the prerequisite mathematical knowledge will not determine the success. Various classroom management practices can be applied from individual seatwork through learning-in-pairs to group work. The students may find this task as an opportunity to show their creativity and how they can apply different parts of their mathematical knowledge in real-life situations.

A mathematical morning (M 10-14)

A possible lesson plan may extend through five stages:

1. Collecting questions;
2. Obtaining information and computing;
3. Designing posters;
4. Presenting posters in a gallery walk; and
5. Evaluating the performance by the students themselves, their classmates and the teacher.

## Classroom practice

At the beginning of the lesson, we give a motivating introduction:
'Today we will work on a task that is built on your everyday experiences. First, please write down what things you usually do in the morning until you arrive at school.'

Then students work in pairs with the instruction 'discuss with your partner what questions of mathematical nature you can raise about the morning procedures. Wear "mathematical glasses", and try to ask mathematically whatever you want to know'.

The pairs' ideas are then written on the blackboard. They can decide whether any of them is a mathematical question at all.

Then students' work in groups, choosing two mathematical questions that have already been written on the blackboard. Make sure you have equipment available to help students with measurements and calculations needed to find the answers to their questions. In case students have difficulties, we encourage them in a way like 'what do you think about travelling?' or 'what if there is a power cut in the morning' etc. Always avoid judging their selection or thinking processes, and try to encourage their divergent and creative thinking.

At the end of the lesson or — if the mathematical morning task was the impulse for a whole series of lessons — at the end of the sequence, students present their questions and results to their classmates. We also encourage them to reflect on the task solving process, and what they had found to be the most difficult step to make. Read some of the students' comments:

- This toothpaste task was so neat!
- Now I see how one can measure the amount of tap water during a given time frame by using a bottle.
- These tasks have been so interesting!
- Shall we learn everything in the future in this way?

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**Classroom experience and photos:**
Zoltánné Sápi, Hungary; Patrick Bronner, Germany
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1113/Construct+your+own+mathematical+morning/view.do?lang=en







Mathematics · Ages: 10-14

# More coffee or more milk?

How would teachers and students fancy starting the morning mathematics class with coffee and milk? Only the croissants are missing.
The coffee-milk-problem offers students the chance to reflect on the modelling of a very simple, yet quite puzzling situation:

> Two identical glasses are filled respectively with the same quantity of coffee and milk.
>
> With a spoon, one takes some coffee from the first glass and pours it into the glass of milk and mixes it with the spoon.
> Then, with the same spoon, one takes exactly the same quantity as before from the glass containing the mixture of milk and coffee and pours it into the glass of coffee, then mixes it.
>
> Which quantity is more? The coffee in the glass of milk, or the milk in the glass of coffee?

**Class time needed: 2 lessons**

## Pedagogical issues

Wine and water can be used instead of milk and coffee (see **PRIMAS website**). An extension can be discussed with oil and vinegar. This is a very open activity.

This situation presents a problem in which no mathematical signs are present at first sight. The main goal for students is to select what kind of mathematical tools and methods can be relevant to tackle the problem. (See **Professional Development Module 2: Tackling unstructured problems**.) Many students will at first think that there is more coffee in the glass of milk than milk in the glass of coffee, but this first opinion is rapidly discredited by a new glance at the situation and mental experiments. However, this first qualitative approach needs to be completed by some calculations and models (graphical, algebraic, etc.) We can use this situation to introduce students to key questions about mathematical modelling. Moreover, it can motivate the change of viewpoint from arithmetic to algebra and also provide a good opportunity for us to discuss key issues about proving in mathematics, such as the use of graphical arguments, the value of one example versus a general proof, etc. A solution with letters and algebra is the most general way to prove entirely and rigorously the result. Nevertheless, a short but sophisticated argument with no computation is the most elegant way to answer. A debate with students offers an opportunity to produce, discuss and compare a variety of mathematical arguments, while important issues about modelling can be raised and tackled.

We need to be aware of the different issues that can be approached and worked on with this problem. The students must be left time to develop their own methods. It is important to let them discuss the various solutions and we shouldn't try to impose the algebraic solution too quickly. This way, students can discuss among themselves the validity of different approaches and the corresponding value, advantages, disadvantages, etc. of the methods considered. The problem is an opportunity for a debate in maths class on key issues about modelling and proving in mathematics.

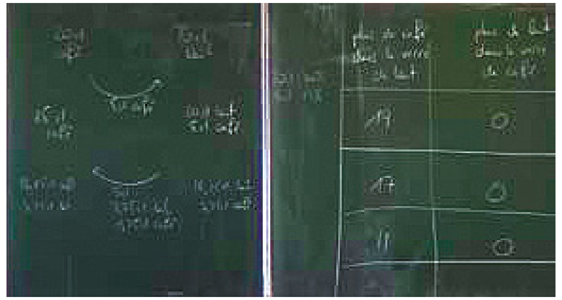
## Classroom practice

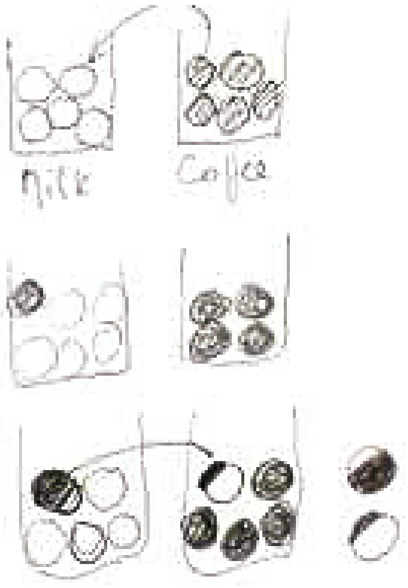
In a first phase, give students ten minutes to form their own opinions about the result. Then we call for an open vote and write the results on the blackboard according to four types of answers: 'more coffee' / 'more milk' / 'other' / '?'. Then in a second phase, students are asked to write a letter that tries to convince one of their friends that their solution to the problem is correct. Writing a letter is a means to ascertain and sharpen one's own thoughts. After 15-20 minutes of thinking and writing, students are asked to vote again (usually there are changes). Then a debate will take place. Students are asked to present their arguments and different approaches to solving the task. Others will oppose them with differing arguments. At the end of the lesson, we should lead the discussion to the evaluation of the various students' solutions and summarise the main results.

In students' experimentations with this task, we observed several approaches corresponding to different mathematical models for solving the task:

• **Numerical models.** The quantities of liquid in the glass and in the spoon can be specified by numerical values. For instance the quantity of liquid in the spoon can be expressed either by a measure or as a proportion (or percentage) of the quantity of liquid in the glass. These examples, if correctly computed, lead to the correct answer, i.e. both quantities, of milk in the coffee and coffee in the milk, are equal. Nevertheless, the difficulties inherent to calculations in this context may lead to a wrong answer, and students being convinced that there is more coffee than milk can unconsciously distort their calculations in order to prove their conviction.

• **Graphical models.** One can draw glasses and represent the liquids in it at the different stages, by cutting the content in the glass in different proportions. This leads to a more or less sophisticated graphical proof. A formal version of a graphical model appeared in several of our experimentations. The students had replaced the liquid with balls of two different colours in such a way that calculations were easy to make.










More coffee or more milk? (M 10-14 or 15-18)

• **Models using letters.** These can be mixed with numerical models, or even graphical models. Using letters to designate objects is often seen as a mathematical ability. Here, the use of letters is efficient when applied to the unknown initial quantities of liquid in the glasses and in the spoon (the parameters of the situation). The latter can be expressed either independently or as a proportion of the first. Below, we give a succinct proof, using Q as the quantity of coffee and milk in the glasses at the beginning and q as the quantity of liquid transferred with the spoon. The following table shows the quantity of each liquid in each glass at the three stages of the situation:

• **Extreme cases.** One can imagine that the spoon is as big as the glass (i.e. the first spoon empties the glass of coffee). It is then easy to see that, at the end, each glass contains half milk and half coffee. On the other hand, one can imagine that the spoon is empty, in which case, the contents of the glasses remain identical. These two extreme cases are unrealistic and lead to the right conclusion without much calculation. A student offering such an argument is very likely to have a good understanding of the power of modelling for the situation.

| | Milk in glass A | Coffee in glass A | Milk in glass B | Coffee in glass B |
|---|---|---|---|---|
| Stage 0 | 0 | Q | Q | 0 |
| Stage 1 | 0 | Q-q | Q | q |
| Stage 2 | Qq/(Q+q) | Q2/(Q+q) | Q2/(Q+q) | Qq/(Q+q) |

**Author:**
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**Photos:**
Isabelle Descombes
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1066/The+wine+water+situation/view.do?lang=en



Mathematics · Ages: 10-14 or 15-18

# Conditions of germination

In this task, students do not carry out an actual experiment. Instead, they practice the steps of the inquiry process starting from a given data collection, namely students' records which contain the results of germination experiments performed under different conditions. The task is to find out the internal and external conditions of germination through analysing these records. Students work in groups and are guided by research questions.

**Class time needed: one or two lessons**

## Pedagogical issues

The task is applicable when teaching the topic of plant development. Students should have pre-knowledge of plant growth, development, fruit and seeds and seed parts.

The students' reasoning and creativity is required during the processes of identifying methods of data analysis, referring one piece of data to another, drawing conclusions from the results of specific examples, discovering relationships and connections, and formulating general observations. This is a type of inductive learning. For the students, records of the cases and the aspects they present may serve as an example of acquiring knowledge of scientific research methods and demonstrate the need for accurate and precise note-taking involved in scientific research.

We play an important role in the formulation of research questions, in ensuring appropriate conditions for students' activity, in facilitating the group work and in guiding the formulation of general conclusions and regularities. Especially for the phase of finding and evaluating possible research questions, look for support in the PRIMAS Professional Development Module 1: Student-led inquiry.





In the beginning of the lesson, the students are presented with a description of the situation and five records of the germination experiments, like the one in the example below.

| Peti's report book : | |
|---|---|
| 1. Where did you get the seeds? | I bought marrow seeds in the shop. In our garden I could choose a large cucumber. I got the pea seeds from my mom, because she collected some last year. And upstairs in the loft we had some wheat seeds. |
| 2. How did you get out the seeds from the fruit? | I only had to remove seeds from the cucumber. I scratched them out with a spoon and chose a quite large one. |
| 3. How did you prepare the seeds for germination and where did you put the prepared seeds? | I watered a piece of cloth and cut four little pieces out of it. I put the seeds on the cloth pieces. I put these pieces into yogurt cups and put them at the window. The one with the pea seed was dried because I forgot to put water on it. |
| 4. What did you observe after 6 days? | The marrow and the wheat came out, but the pea and the cucumber didn`t sprout (germinate). |

Here are some examples of guiding questions for starting the research process:

What could be the reasons that the seeds didn't germinate in all cases?
Collect and generalise the conditions that are necessary for the germination process.
Compare your observations with those in biology books.
Find out why some crops contain germination inhibitors.
Formulate more questions for further investigation.

There are two ways of carrying out the task. In the first method, students work heterogeneously in groups of four or five. Here, each group analyses only one of the five report books. Conclusions are drawn during a class discussion. The second way of doing the task needs two class lessons. Here, students do homogeneous group work, with each group analysing all five books. Discussion and drawing conclusions take place within the group work phase. This is followed by groups reporting to the entire class. Finally, the class as a whole compares the conclusions and the formulations of generalisations.




Science · Ages: 10-14

## Classroom practice

The task was tried out using the homogeneous group work method during a double lesson (2 x 45 minutes). One student from each group talked about the group's solution pattern. Discussing these solution patterns and strategies took place in the second part of the session. The groups chose quite different solution strategies. One group prepared a table summarising the variables; another used marker pens to highlight the relevant information. A third group divided the task into parts, and students in it undertook specialised roles. The analysis of the measures of the variables and the formulation of the conclusions were not easy to do. The students recognised the effect of water and the ability of germination of the seed more easily. The role of light, temperature and germination retarder was answered only after examining the results of more record books. This latter was successful only with teacher guidance. In order to answer the questions for further investigation, the students used biological lexicons and popular science books provided by the teacher and/or browsed the internet. It was obvious that students enjoyed the inquiry process.

During the task solving process, several everyday problems arose from their experiences or from stories heard from parents and grandparents. They tried to answer these questions amongst themselves, and these questions were re-addressed after discussing the task. This served to help refine the answers they had already given to each other.

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# Iron swings

This unstructured task may be used to develop concepts related to the rusting of iron. Through collaborative work, students choose conditions that are likely to lead to rusting and then plan and carry out an investigation. If the task is used before the conditions for rusting are discussed in class, students will use their real-life experiences of rusting and test their ideas. This task provides an opportunity for students to explore the situation, plan and experiment systematically and interpret and evaluate results.

**Class time needed: three lessons**

## Pedagogical issues

The task provides students with an opportunity to discuss, plan, observe, explain, evaluate, predict, communicate and take more active control of their own learning. Our role is to facilitate group interaction and to encourage students to share and validate their ideas.

The students are not expected to have learnt the conditions required for rusting before doing this task. Ideally, though, they should be familiar with the composition of air and with properties of water as a solvent (including a solvent of oxygen in the air). If students do not possess this prior knowledge, it will emerge as part of the discussion of the results.

A possible lesson plan may extend through three sessions: A planning session; a hands-on session to prepare the experiment; and a post-session (a week or so later) for observing, interpreting and evaluating results.

## Classroom practice

In the first session the problem is presented to the class:

> New swings were placed in playing fields around the country to replace old ones. One set of swings was placed in a playing field by the sea, while another set of swings was placed in a sunny playing field in a town away from the sea. The swings are made of iron. Iron rusts.
> Will the iron in both sets of swings rust? What makes iron rust? Plan an experiment you can do in the laboratory to find out what is needed for rusting to occur.

Students work in groups and discuss the problem. They are encouraged to use their experience and prior knowledge to form hypotheses. This is followed by a whole class discussion. Our role is to collect views and ask students to explain their reasoning. At this point, it is best not to comment or judge answers but only receive their views and encourage clarification and discussion. Main points, such as what makes iron rust, may be listed on the board for all to see.
Possible answers may include:

- Swings in the playing field by the sea will rust but not those in the other playing field.
- Swings in both playing fields will rust, but one will rust faster.
- Swings in both playing fields will rust to the same extent.
- Only water is required for rusting.
- Salt or spray from the sea causes rusting.







At this point, we acknowledge that there are different views and invite students to think: How are we going to find out who is right?
Students go back to work in groups and try to come up with ways of identifying the requirements for rusting. Ideally, they should come up with a way of controlling variables (air only present; water only present; both air and water present; salty water); ways of ensuring that variables are controlled (how to exclude air or how to exclude moisture); source of iron they will use; apparatus; how long they will wait before making observations. If students do not include all variables or if they do not take one variable at a time, we should, ideally, not interfere. Students often suggest that water alone is needed for rusting and that an object submerged in water is only in contact with water. We may ask questions that encourage students to reconsider, for example to think about water as a solvent.
In the second session, students are provided with any apparatus and materials their plans indicate are required. They prepare the experiment. If students do not include all variables, we may opt to prepare our own set of experiments to show students as part of the final discussion.
In the third session, students finally observe the outcome of the experiment and discuss in groups: What makes iron rust? Will the swings in both playing fields rust?

Encourage students to explain their reasoning in a small group. Follow-up with a class discussion where each group reports findings and conclusions. At this stage, we may challenge students' ideas and conclusions. We might compare the conditions they are claiming to use with those present in practice (water that has been standing in air is really water plus air). Alternatively, show the results of an experiment using conditions that the students did not think of (iron submerged in boiled water with a layer of oil on top to prevent contact with air; iron in dry air). Students evaluate the methods they have employed and suggest improvements. The session can conclude by discussing rust prevention based on conclusions drawn from the experiment.

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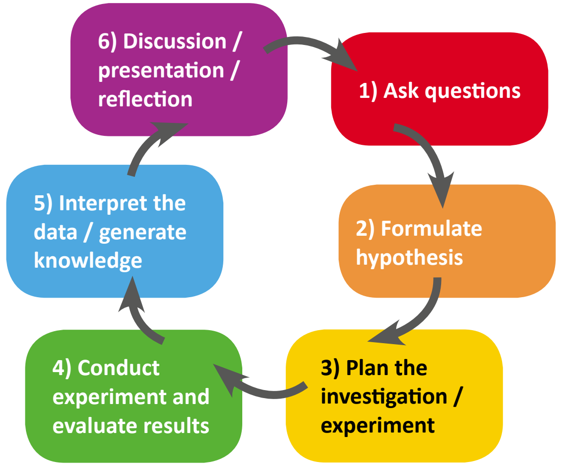
# The candle experiment

The candle experiment is a classic used in natural science classrooms from kindergarten through secondary school levels. Secondary school students are able to answer the original question posed without even conducting the actual experiment: ‘The candle uses up the available oxygen – the air pressure falls – the water rises.’ However, the following experiment that uses ice tea as the liquid, shows clearly that the common explanation is wrong and that students have to work experimentally to find the right explanation.

**Class time needed: two lessons**

## Pedagogical issues

To ensure that a structured approach is used during the independent group work on this open task, it is a good idea to use the research circle as orientation. The research circle can be used with all experimental research questions. In this way, the often muddled hypotheses that students come up with must be clearly formulated and discussed in the group and corresponding experiments exactly planned before they are conducted. Students can be provided with a worksheet that outlines the research circle structure








The candle experiment (S 10-14 or 15-18)

Technically seen, various factors play a role in making the ice tea rise. The most decisive one, though, is the thermal effect: The burning candle warms the air in the tea glass and the air expands. Lack of oxygen makes the candle go out. The air then cools down rapidly and contracts. This creates a vacuum which allows the ice tea to flow into the glass. The higher the difference in temperature, the greater the difference in pressure and the corresponding increase in the level of tea (liquid). This experiment also works without the candle: The glass is first heated with a hot air gun and then placed upside down in a small bowl. The glass is cooled from the outside with ice cubes. If students observe carefully while experimenting, they will also discover other relevant factors, such as condensation on the cooled tea glass. Depending on lesson time available, the research circle can be used to do further experimenting with the students’ observations. Here, interim results can be presented and discussed.

The thermal factors that students have researched on their own can be transferred to numerous thermodynamics applications that all function according to the ‘No go without hot and cold’ principle of thermal engines. Examples of this include: the expansion of air in an Erlenmeyer flask that is alternately immersed in hot and then cold water whilst attached to a gas syringe; Stirling Engine motor movement generated only by applying heat on one side (the palm of a hand) and cold (ice cubes) on the other; the classic steam engine with its hot side (fire) and cold side (condenser); a coal-fired power plant located near a river for cooling, and an automobile motor with water-cooling system.

## Classroom practice

To help motivate the class and introduce the task, we, as teachers, might tell students a story about a little accident we had: ‘Ice tea spilt during a date! How can I save the situation and get the ice tea back in the glass? With magic and a candle!’

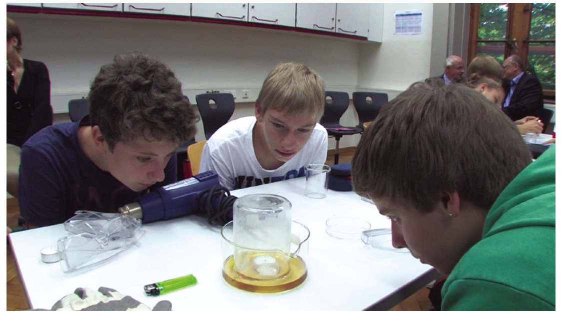
Secondary school students in year nine immediately gave an explanation for the ‘magic’ trick: ‘The candle uses up the oxygen – air pressure in the glass decreases – the ice tea rises.’ When asked the logical question, ‘when does the ice tea rise the most?’, students answered clearly with, ‘while the candle is burning.’ However, careful observation reveals that the liquid only starts rising when the candle flame begins decreasing. Therefore, depleted oxygen can be eliminated as the cause. This is when things really start getting interesting for the students: ‘Magic – or is it really physics? What makes the ice tea rise?’



Science · Ages: 10-14 or 15-18

For researching this phenomenon, we divided the class into groups. All kinds of materials that could be used for planning and conducting the candle experiment were made available on a large table in the classroom: Support stands, magnets, salt, hot air gun, sugar, weights, indicator paper... Alternatively, we can have the materials the students need ready and available for the next lesson. It is surprising how creative some groups were when planning their experiment, discussing in the team their preconceptions and thereby, including their knowledge of all the natural sciences.

An eight minute video with English sub-titles that shows part of this lesson is available at the following link: http://www.youtube.com/watch?v=BSDK7KjcT6k.



**Author and Photos:**
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**Link to Primas website:**
http://www.primas-project.eu/artikel/en/1524/The+candle+experiment+/view.do?lang=en







# Define it!

Defining a concept requires introducing all aspects necessary to cover the concept as a whole without including unnecessary information. For students, it is helpful when we give them examples and non-examples of the concept. Students are better able to learn how to build concepts by making their own attempts, rather than through our input.

In this activity, students work in groups of about four. First, they think for a few minutes about the given concept and try to define it. Then they should discuss their ideas in the group. This can take about 15 minutes when working on a couple of concepts. Afterwards, initiate a classroom discussion to bring it all together and come to proper and commonly accepted definitions. (Think-Pair-Share method.)

**Class time needed: part of one lesson**

**Plant**

Giv
tha

**Element**

Giv
tha

**Weight**

Give a definition for the concept 'weight' that is complete and as short as possible.

- Is all the content of your definition sufficient and necessary?
- Give examples and non-examples that clarify the definition.

## Pedagogical issues

In traditional teaching, we define a concept, and the students are supposed to use, but not discuss it. Often students build their own (not necessarily correct) ideas of the concept without even being aware that they are doing so. In the introduced activity, students are motivated to express their thoughts about the concepts and communicate their ideas with others.

**Parallelogram**

Is the following definition true for a parallelogram?
Is all the content of the definition sufficient and necessary?

**A parallelogram is a quadrangle where the diagonals cut each other in half.**

They have to be critical about what other students think and about their own ideas. While talking with each other, the students enrich and refine their idea of the concept. When using this activity as an introduction to a lesson about new concepts, students will activate their prior knowledge and can mirror it to the knowledge of others. We can walk around and listen to what the students are telling each other and get a good idea of their knowledge of the concepts. The activity has been used in Professional Development Module 3: Learning concepts through IBL. It can easily be adapted to the normal classroom practice, as it requires no special equipment and takes only part of a lesson.

## Classroom practice

A science teacher gave the task to define the concepts ‘resistance’, ‘conduction’ and ‘insulation’ to groups of students at the beginning of a lesson. In the following, the students did experiments to find out if a material was a conductor or an insulator.
At first, students actively discussed the concepts. During the experiment, they investigated with the clear aim of discovering the differences between the materials (conductors, insulators and resistors.) After the lesson, it was clear that they had obtained a good idea about the concepts conduction and insulation. However, contrary to the teacher’s hopes, they had not used the concept of resistance to distinguish a conductor from an insulator.
Nevertheless, the teacher was enthusiastic about the activities, observing that throughout the entire lesson, all students were active and involved. As a result, students had improved their understanding of some central physics concepts.

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**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1605/Talking+about+definitions/view.do?lang=en





# Algebra pyramids

The algebraic expressions in the cells of the pyramid are found by multiplying the expressions of two adjacent cells directly below that cell, as shown in the left pyramid:

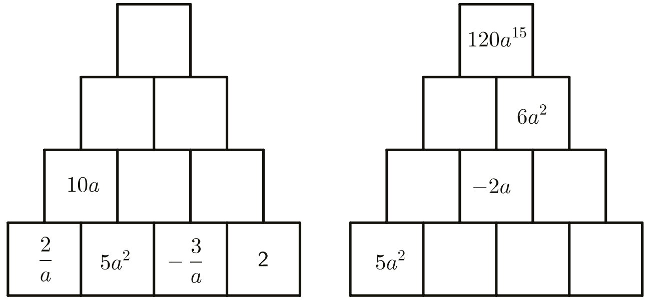
$$\frac{2}{a} \times 5a^2 = 10\,a$$

The left pyramid presents a straightforward task that goes one way from the bottom up to the top of the pyramid.
The right pyramid asks for strategies that are not at all clear from the beginning.

**Class time needed: (part of) one lesson.**

**Two multiplication-pyramids**

Task 1: Find the missing expressions.




Task 2: Design a simple and complex pyramid that can be solved by someone else. Your design will be used by others for practicing algebra skills.

## Pedagogical issues

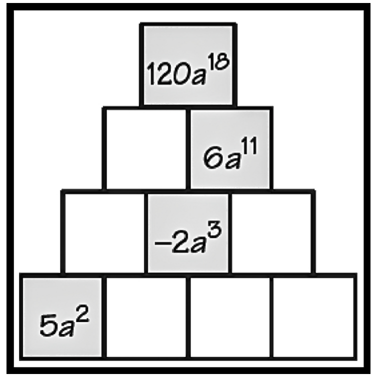
Practicing algebraic skills is mainly seen as working through a set of reproduction exercises based on worked out examples in the textbook and/or examples we give. These often go from simple to more complex. A quite simple variation on these standard exercises can change a classroom setting from doing standard exercises with prescribed, correct answers into a dynamic environment in which investigations, discussions, problem solving activities and students’ creativity come up very naturally. These problems are productive rather than non-productive exercises.
A collection of these kinds of productive algebra tasks we can use for practicing many aspects of algebraic skills can be found in a unit designed by Martin Kindt (Freudenthal Institute, 2004): Positive Algebra: a collection of productive exercises.

The activity as presented above was used in a classroom after a session on Tackling unstructured problems, the Professional Development Module 2 on the PRIMAS website. The second task in the activity is an example that illustrates how students’ own productions can help to make clear what level of conceptual and procedural knowledge students have already obtained and build on what they know. Professional Development Modules 3 and 6 aim at stimulating teachers' ability to design and try out similar tasks in their classrooms. (See Professional Development Modules 3: Learning concepts through IBL, and 6: Building on what students already know.)

Apart from practicing algebra skills, this kind of task requires the student skills of problem solving, choosing own strategies and communicating ideas and results. As part of the inquiry process, students need (individually or in small group settings) to explore, investigate and communicate their ways of working on the given task.

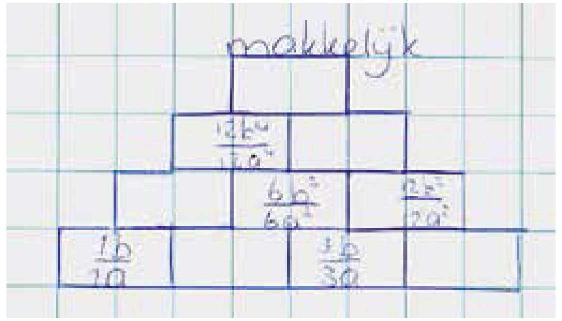
## Classroom practice

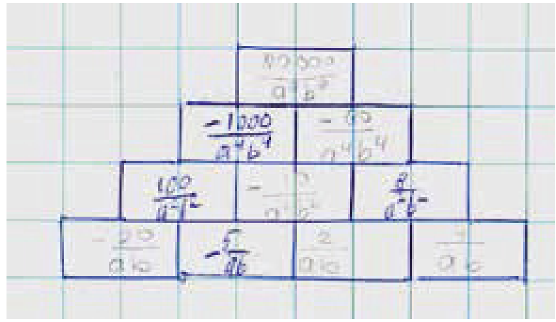
A Mathematics teacher took two textbook exercises that were simple versions of the algebra pyramid and devised an unstructured version of them. She first asked her students to analyse the following pyramid and try to discover how it is constructed:




Students were expected to recognise that, from the bottom up, two adjacent sections are multiplied and the outcome is written in the square above. They had recently been working on multiplying variables, so this was not unreasonable.

After five minutes and some discussion, she challenged the students by asking them to design such pyramids themselves that were to be solved by fellow students. One 'easy' ('makkelijk' below) and one more 'complex' pyramid that came up in the classroom are presented here:









First of all, the teacher was amazed by the students' engagement while working with algebraic expressions for more than 30 minutes. During an intermediate classroom discussion, one pair posed the problem of the minimal amount of information that must be supplied for such a pyramid to be solved. This inquiry question was taken up by other groups. The teacher was enthusiastic, and reported that the students improved their understanding of algebraic skills more than usual. She commented that the students 'owned' the mathematical problems they had solved.

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**Photos:**
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**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1526/An+algebra+pyramid/view.do?lang=en

# Beads and formulas

This task and activity is about teaching Newton's binomial theorem and some related concepts (combinations, Pascal's triangle) by using an IBL activity based on modelling the spread of rumours. One of the main strengths of this approach is that handmade models can be created by using some plastic beads and fishing lines (or other strings) in order to emphasise not only the number of combinations of k elements that can be chosen from an n-element set, but also the combinations themselves. The inquiry-based approach is highly suitable in developing the model, formulating basic properties of the combinations and in developing more general models.

**Class time needed: 2 to 4 lessons depending on the depth of inquiry**



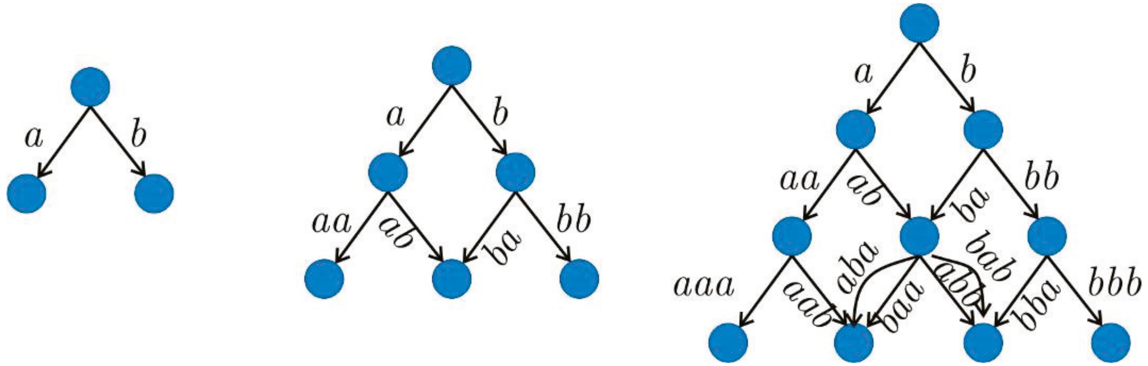
## Pedagogical issues

Modelling in this context does not mean aiming at a faithful representation of extra-mathematical reality, but it is more likely an intelligent game with elements of reality, following strict rules. This should also be made clear to students.

The scenario is as follows: A particular school is famous for the amount of gossiping that goes on there. A school boy hears a rumour about one of the teachers, and he can't stop himself spreading the rumour the next day. First of all, he exaggerates the rumour (i.e. amplifies it by *a* times), and tells someone. Then he exaggerates it even more (*b* times) and tells someone else that version. But this is a very gossipy school – so everyone who hears a version of the rumour, does the same. The main task is to organise clubs in this school so that each club invites those who heard the rumour with the same amplification factor. So we have to invite to the same club those who heard the rumour after amplification with *a* and *b* and those who heard the amplified version by *b* and *a* but no other student will be invited to this club (cf. scheme of the first three rounds of gossiping, the balls on the lower row representing the clubs on this level).





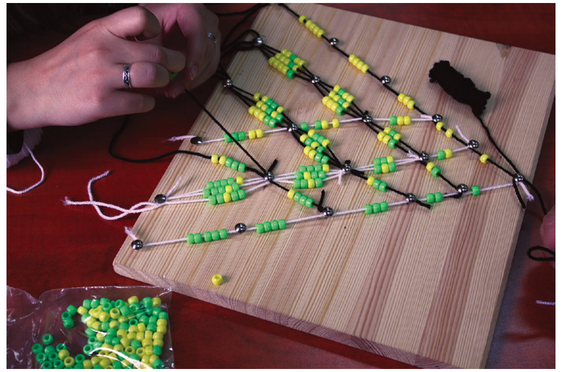



As a first step, it is acceptable to use a pen and paper approach where students try to illustrate or present that their founding is acceptable. However, in order to understand the deeper structure of rumour spreading, we recommend asking students to make handmade models using some strings and beads in order to clarify some aspects related to the combinations, to variations with repetitions and the ways they can be generated (by a computer program or by using this handmade model).

After this step, we can organise some concluding briefings where we define the mathematical concepts involved and formulate the properties students found using standard mathematical language.

Further investigation may start from the question: What happens if each student tells three (or more) different versions to other students? If we want to construct a handmade model for this case (some kind of Pascal tetrahedron), we need a special session for this because the volume of work increases exponentially. Our experience is that students can develop their pen and paper models within one hour. For the handmade models they need more time (at least 1.5 - 2 hours) and for doing the 3D models (the Pascal tetrahedrons), they require significantly more time. For this reason, we can organise this last step as project work. Further details about the practical aspects of this activity are given on the PRIMAS website.

This activity is exemplary for changing the classical approach of a well-defined curriculum content into a complex IBL activity.



## Classroom practice

The most common first approach of students was to model the spread of rumour by a binary tree. In some cases, they discovered that this binary tree can be transformed into the Pascal triangle through identifying (joining) the vertices that correspond to the same factor of enlargement. This also means a slightly different viewpoint because in the binary tree, the vertices correspond to individual students, while in the transformed configuration, the vertices correspond to the clubs.

By using the strings and beads, this transformation can easily be seen. The final model also shows the combinations (variations with repetitions) that correspond to each node (see photo). Two of the most important outcomes of this activity were: We could eliminate from the very beginning common misconceptions regarding combinations (we did not work with the usual formula at this level) and students could develop on an intuitive level the steps of formal proofs for several properties (e.g. the rule for generating the Pascal triangle and the binomial theorem). By constructing different material-based models within the environment of a reality-related game (i.e. the situation of spreading rumours), students were able to generate a concrete conception of a complex algebraic or stochastic content.

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**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1583/Beads+and+formulas/view.do?lang=en



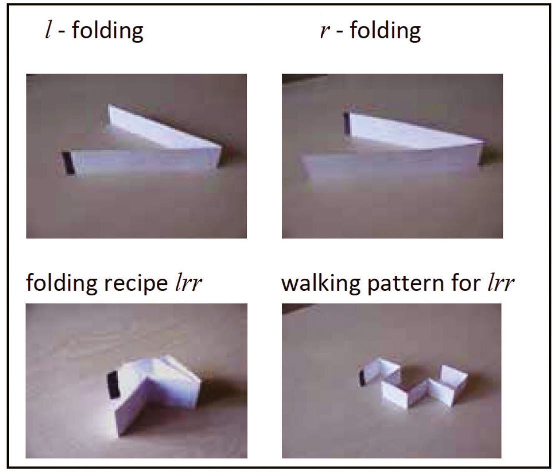




# (Cr)easy! Paper strip folding

Simple strips of folded paper are the key to this task that plunges students into the world of working mathematically. In it, a paper strip is folded several times. The starting point is indicated with a small mark. There are two ways of folding:

l - The end of the strip is behind the starting point.
r - The end of the strip is in front of the starting point.




**A folding recipe** is a series of letters l and r. After unfolding you see a walking pattern. A walking pattern can be described by R (right turn) or L (left turn) when walking from starting point to end point. The walking pattern that fits the recipe lrr is RRLLRLL (see pictures). Three times folding results in a walking pattern of eight straight parts and seven turns.

Intriguing questions: Can you predict the walking pattern from any folding recipe? Why are starting and end parts of the pattern always perpendicular? There are $2^n$ different walking patterns after n foldings. All patterns have the same distance between starting point and end point. Why?

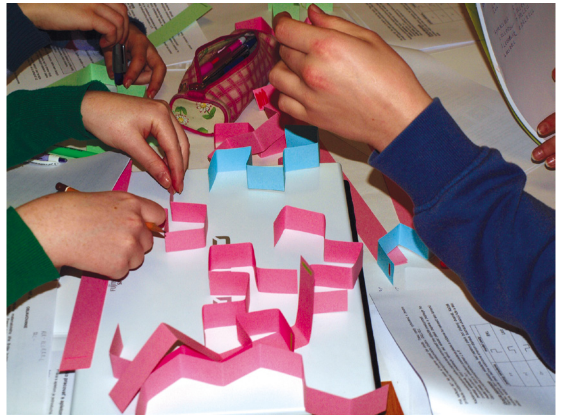
**Class time needed: one day (7-8 hours) or a sequence of lessons**

## Pedagogical issues and mathematics contest experience

This challenging task is ideal for working with gifted young people in math clubs or at project days or – as described here – in a mathematical contest (like Mathematics B-day in the Netherlands and Slovakia): ‘Students are challenged to show process skills in developing strategies, making conjectures, trying to prove or reject these, logical reasoning, critically reviewing and adjusting models and in cooperating.’ (http://www.fisme.science.uu.nl/fisme/en/projects/indexorg.php)

During the day of the contest (Mathematics B-day Slovakia 2012), more than one hundred students were observed discovering different ways of working mathematically. First of all, they had to read and understand the assignment (a complex mathematical text). During group work, students went through all the processes of inquiry based learning: exploring the situation, experimenting systematically, communicating and evaluating results.

The outcome of an entire day of intense, mathematical work was a report about the investigation process, problem solving results and interpretations.



Hands-on activities.

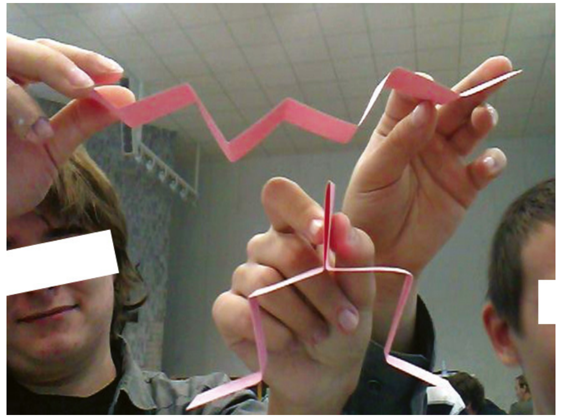
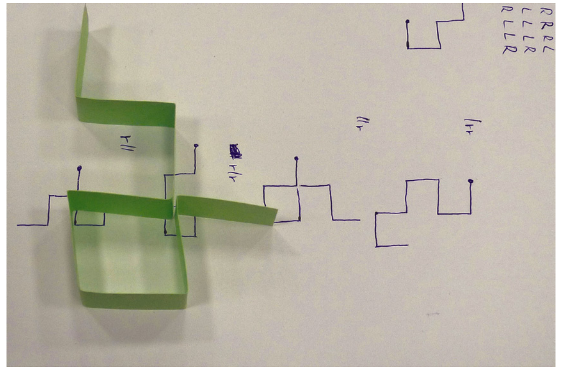


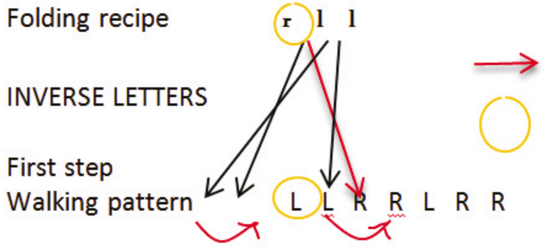
Illustration to the answer of the exploratory question: You won’t find the zigzag pattern as a possible three-fold pattern. Could you have known this in advance. How?



(Cr)easy! Paper strip folding (M 15-18)



Walking pattern and its graphical record.




Folding recipe and corresponding walking pattern relationship.

Paper was all over the place in the classrooms during the first hours of the contest. Several groups of three - four students found original solutions to the problems posed and described their arguments precisely. Some teams added original pictures, photographs and/or illustrations of their findings. One group even wrote a fairytale about paper folding with very original comments and challenges for the reader. At the end of the day, one of the participants stated:

*‘It was the most interesting day with mathematics in my life so far. Thank you for this wonderful experience!’*

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**Photos:**
Henk van der Kooij, Sona Ceretkova, Janka Melusova, Katka Marcekova

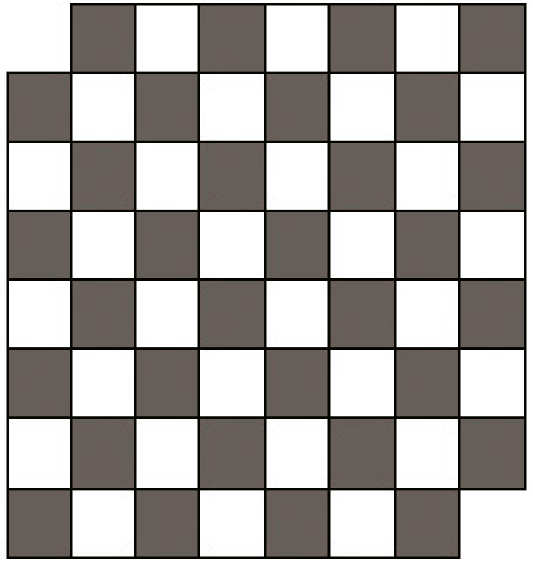
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1682/Paper+strip+folding/view.do?lang=en
**Background information and reports about Mathematics B-day:**
http://www.primas-project.eu/artikel/en/1137/Mathematics+B-day/view.do?lang=en



Mathematics · Ages: 15-18

# Playing with tiles and colours

**Suppose a standard 8x8 chessboard has two diagonally opposite corners removed, leaving 62 squares. Is it possible to place 31 dominoes of size 2x1 so as to cover all of these squares?**



This activity builds on the famous mutilated chessboard problem originally proposed by Max Black and offers students opportunities to explore variations of this and other famous, mathematical problems related to the Parity Problem. It further provides solutions in the 'conceptual sense', based on tiles and colours, without formal proofs.

**Class time needed: Two 45-minute lessons**

## Pedagogical issues

The mutilated chessboard problem is a tiling puzzle proposed by philosopher Max Black in his book 'Critical Thinking' (1946). It was later discussed by Martin Gardner in his Scientific American column 'Mathematical Games'. This activity uses the problem: 'Suppose a standard 8x8 chessboard has two diagonally opposite corners removed, leaving 62 squares. Is it possible to place 31 dominoes of size 2x1 so as to cover all of these squares?' The task provides opportunities for students to formulate 'conceptual proofs' while using tiles and different colours (coloured dominoes).

The activity is designed for two, forty-five minute sessions. The first deals with the problem proposed by Black and some of its variations. The second session focuses on an extension of the problem. During the beginning of the first session, students can explore an easier version of the original problem:

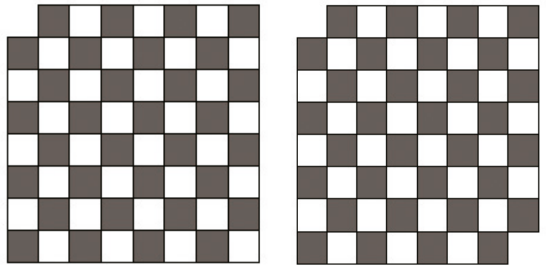




Playing with tiles and colours (M 15-18 )

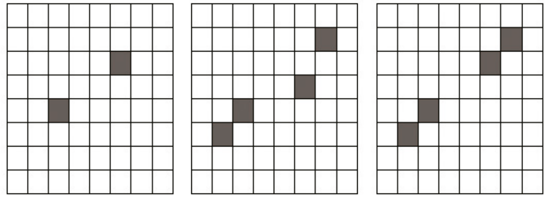
**Is it possible to tile an 8x8 chessboard using 1x2 dominoes?**

We, as teachers, should note that it is essential to use dominoes of different colours; otherwise the students will not be able to reach a solution (at least easily and without a formal proof). Using dominoes, like the one presented above, students can make connections between the chessboard and the dominoes they have to use to cover the board. Starting from an easier version of the problem, we can assist students to discover that each domino covers one white square and one grey square. So 31 dominoes would cover 31 white and 31 grey squares. So the task is possible. Students can then move to two other problems: 'Is it possible to tile the 8x8 chessboard using 1x2 dominoes if: (a) a corner square is removed? (b) The two opposite corner squares are removed?' Using the same approach, students can 'prove' that this is impossible, because in the first case, one square will be left (either grey or white), while in the second case, cutting two opposite corners will result in removing two squares of the same colour.



In the second session, students can extend their findings in solving other variations of the problem, like those presented below, and examine whether it is possible to tile the chessboard if some squares are removed (grey squares in images below). Students can determine their solutions with a *'more general colour proof'*.

During the whole process of problem solving we, as teachers, should put particular emphasis on the dialogue that aims to build a shared mathematical understanding based on the students' experimental results.





Mathematics · Ages: 15-18

## Classroom practice

In an implementation of the activity in a year 11 classroom (16 year olds), students were really engaged in the activity. They reported that they enjoyed it very much, especially because it was different than usual mathematics and formal proofs.

Students working in groups of four managed to solve the majority of the provided problems. Some students, moving a step further, created some chessboards with missing squares (based on the examples provided), and identified whether these boards could be covered by dominoes, or not. Students' engagement in posing interesting and challenging problems was facilitated by the teachers' appropriate hints and prompts. Finally, one group of students extended their explorations (following teacher's suggestion) in using square and L-shape tetrominoes to tile a chessboard.

**Author and photos:**
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**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1681/Playing+with+tiles+and+colours/view.do?lang=en





# Keep it cold!

This task offers an opportunity to facilitate more motivating and meaningful science learning. It starts from students' preconceptions about the thermal properties of some common materials and engages them in a hands and mind-on investigation process as students search for evidence to base their decisions.

The qualitative analysis of students' arguments before and after the implementation of the activity has proven its efficiency for promoting highly engaging and meaningful learning about the thermal properties of some common materials.

It is a classroom activity for level one secondary school that can be easily integrated into daily teaching using readily available materials (ice cubes, metal and plastic containers, wool…)

**Class time needed: 2-3 lessons**

## Pedagogical issues

The activity may be introduced as a challenge to capture students' interest from the beginning:

> **'You are involved in a rescue mission and you have to wait for extra help. The outside temperature is hot and you urgently need to keep some medicines cold. However, the only resources available are woolen blankets and some metal containers. What would you do to keep the medicines cold for longer: wrap them in wool or place them in a metal container?'**

Once the situation is introduced, students are asked to work in small groups of three or four to discuss the problem and to decide what they would do. Then they have to justify their decision and make an initial hypothesis about what is the best way to keep the medicines cold (wrapped in wool or inside a metal container). Afterwards, students design an experiment to test their hypothesis. Finally, they must evaluate the adequacy of their initial decision/ideas based on the available evidence.

This activity prompts students to tackle a problem, formulate a hypothesis, design and conduct an experiment to test their hypothesis, interpret the results, and evaluate their previous ideas in light of the experience.

Students are asked to work collaboratively and communicate their final results; therefore negotiation and explanation of ideas play an important role and facilitate the social construction of knowledge.

We should try to promote an atmosphere of confidence where students are encouraged to express and exchange alternative viewpoints and strategies, without right or wrong answers, but as intellectually enriching challenges. It is important that we subtly orientate group discussions. Encourage reasoning rather than getting the answer, and help students to make connections between ideas and extract the important ones.







This activity has also been designed to facilitate the learning of concepts and models to explain the thermal properties of some common materials. Building on students' previous ideas, it offers an opportunity to challenge students' misconceptions and create cognitive conflict, making students reconsider their former ideas and setting the stage for meaningful learning. Finally, students are asked to look for and discuss scientific models to explain their experimental results

## Classroom practice

The following excerpt from a student's explanation of her initial hypothesis, illustrates some of students' misconceptions, which are challenged by the IBL activity.

*'I would keep them in a metal container because wool produces heat and metal remains cold, well, except if you place the metal container under the sun'.*

The implementation of the activity in a level 1 secondary school class in Spain showed that only 27% of students formulated correct hypothesis about the thermal properties of wool and metal. On the contrary, 92% percent of students offered appropriate answers about the thermal properties of these materials after the IBL activity.

The analysis of the teacher's written account of the experience reveals that the activity is highly motivating and engaging. We include some quotes from the teacher's written accounts:

*'The introduction of the activity raised expectancy and excitement among students, since they are not used to being challenged by these kinds of questions so closely connected to their daily experiences, but on which they had not reflected before…'*

*'Contrasting their previous hypothesis by experimenting and having to present and explain their results were activities that engaged the whole class in an interesting and heated debate on energy transfer and thermal conductivity, connecting scientific ideas with students' own experience…'*

These results reveal this classroom activity as a powerful resource to promote active and motivating learning, prompting the development of processing skills, conceptual change, and meaningful science learning.

Similar motivating experiences have been reported by pre-service teachers in Norway who tried out this task with 12 year old students:

*'I have seen the commitment of the students when they are presented with a problem, which inspires me to take this exploratory approach further. It seemed that the students automatically were motivated to find out the solution, almost as if it was a competition'.*

*'We also asked the students if they liked this way of working, and they almost shouted "YES" in unison'.*

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**Photos:**
Daniel Aguirre
**Link to PRIMAS website:**
http://www.primas-project.eu/artikel/en/1360/Keep+it+cold%2C+wool+or+metal/view.do?lang=en







# Shake torch

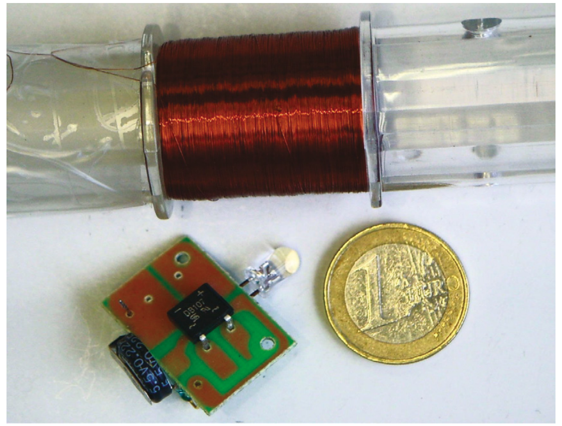
A Faraday torch (shake torch) is known to many students from camping. In addition to its practical functions, the shake torch can also be used to explore the phenomenon of induction. In class, each group of students is provided with a shake torch and a copy of the following question: 'How can such a thing work?' First, the students need to examine the torch components and explore its functions experimentally. The goal here is for students to understand the device so completely that they are able to construct their own Faraday torch.

**Class time needed: 5-10 lessons**

## Pedagogical issues

The technical make-up of a shake torch is not that complicated (coil, magnet, four diodes, capacitor, LED bulb). Level II secondary school students should be able to research and explore it independently while learning about induction. However, the fundamentals of capacitor charging and discharging and diode properties should be covered in advance. Depending on your available class time, facilities and equipment, this task can also be done without having students make their own shake torches.

Researching the physical phenomena related to shake torches is also appropriate for middle



school students. However, here the class will need more guidance when examining the properties involved. This can be accomplished using directed, explorative learning.

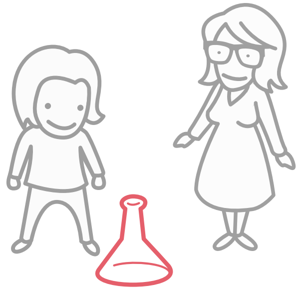
In order for students to conduct their experiments, you will need to make available specially prepared torches, a wide-variety of materials from your physics collection and modern data logging systems.

First, students use the research circle (see page 50: The candle experiment) to examine the functions of the torch. Students generate hypotheses, plan experiments to test these, take and record measurement curves and formulate qualitative physical laws about induction. At the end, students present their results in the form of a gallery walk.

## Classroom practice

The task was given to upper secondary students in a project at the end of the school year. With the knowledge students had gained through their independent work in examining the functions of a Faraday torch, they were able to construct their own shake torch. (A list of needed materials is available on the **PRIMAS homepage**). Students had to wind the coil, solder the rectifier diodes and design an appropriate body for the torch. The guidelines for achieving an optimum final product were: having a good design and achieving the longest possible light duration with the least amount of shaking. The students' results were really worth seeing!

Replica of torch made by a student



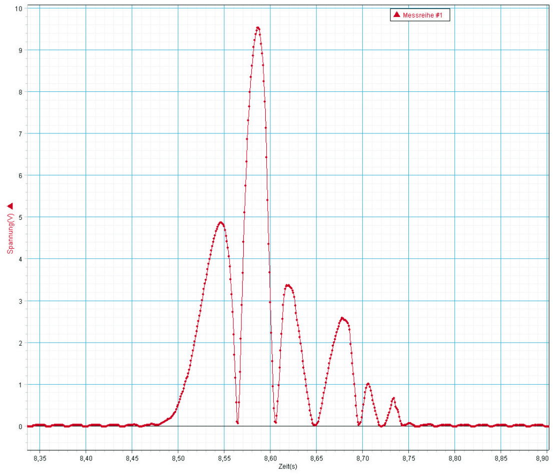




Shake torch (S 15-18)

When working on this project, students exhibited a high level of motivation as the lessons' contents ranged from physical phenomena to induction fundamentals to constructing a technical product on their own. Students were especially interested in comparing and discussing the measurement curves from the original torch to those they had made themselves.

Capacitor charged by shaking, discharged with the white LED lamp.



The magnet falls in the torch through the coil onto the mechanical spring: Measurement after the rectifier.

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**Author and photos:**
Patrick Bronner, Pädagogische Hochschule Freiburg, Germany
**Reference:**
E. Claus et al.: 'Shake Your Light! ...', Unterricht Physik, 23 (2012), 127
**Link to the PRIMAS website:**
http://www.primas-project.eu/artikel/en/1662/Shake+me!/view.do

Inquiry-based learning in day-to-day classes

# Inquiry-based learning in day-to-day classes

Changing the way of teaching and adopting an IBL perspective is an inspiring process in which we can gain a lot of interesting insights. However, there are also several challenges we have to overcome. Let's listen to some teachers' experiences across Europe.

## What teachers say about inquiry-based learning

**Gabriel Abela, teacher from Malta:**

'At first, I was not convinced that IBL would be effective - especially with high-ability students. I was concerned that these students might consider IBL a waste of time and not like it because they are used to explanations followed by many drilling exercises. But when I tried this (new) approach, students liked it.

Regarding the low-ability and the average-ability students, I was expecting them to accept this method readily. These students are talkative – and this method gives them more opportunity to talk to one another. However, there were instances when their discussions were not maths related. Still the method worked with them, because they were working together. In regards to the group work, I make sure that the same four students don't work together in the same group all the time. This way, the students have the opportunity to get to know all their peers.'

**Claudia Matt, teacher from Germany:**

'If students like it, the lessons are much more fun and pass more quickly than usual. Many students don't like maths. But if they can work with IBL, they do have more fun. And they like it even more when we do IBL and consider different areas of study as well, e.g. geography or biology, something students are interested in besides maths. This makes the lessons even more fun, the skills students acquire are much more diverse and maths do not remain so isolated and unrelated to students' lives and interests. We still do struggle with maths not being many students' favourite subject and the sentence, 'I just can't do it', is a one you hear quite often. It took quite a while to teach them that they CAN do maths, but to do so, they need to practice – that's just how it works. There are many exciting things to be discovered. It is a process and it might become complicated if you teach a class you have not known or taught before. For example, it's difficult at secondary school if you start with IBL in a year 8 or 9 class - and those students aren't used to this way of learning. But if you have a 5th year class and slowly introduce IBL, it will, of course, be much easier for the students - and yourself. The later you take over a class and try to introduce IBL, the more difficulties might arise, since you have to start all over again. But like I said, it depends on the class, as well. The one I'm teaching at the moment struggles quite a bit with IBL. This might be due to the classical way of teaching, which was the only strategy they had known before.'

Photo: Patrick Bronner

# Inspiring experiences – and challenges

Many teachers have commented to us that IBL approaches provide a more enjoyable and engaging experience for students:

*'The students are very open to this type of activity. They enjoy these because most of the problems are usually based on real life situations. This is quite different from our usual tasks.'*

(Vadim Mazilescu, teacher from Romania)

*'It's great fun to plan mathematics now. I think differently as I plan. I tend to think more about what students need to understand and how they can benefit from each other's knowledge. I try to find projects where students can use each other and can work in groups.'*

(Susan Gjertsen, teacher from Norway)

*'I also see the unique opportunities that come from implementing such teaching. Exploratory tasks allow students to use their own experience and logical thinking to a greater extent.'*

(Linnea Hakonsen, pre-service teacher from Norway)

These approaches also provide many challenges. Below, we discuss some of these issues and suggest possible ways of responding to them. Our suggestions are supported by reports from teachers that are already implementing IBL.





### 'IBL takes too much time.'

Many teachers claim that they find it difficult to integrate IBL into their existing schemes of work. When it comes to covering syllabus content, they view IBL pedagogies as being less efficient than traditional, textbook approaches:

*'A difficulty is time and the syllabus. We always complain about the syllabus. Our syllabus wasn't designed with IBL in mind. I wish that these activities could be incorporated into the syllabus. If you want to use IBL, you feel that you won't be able to cover everything on the syllabus.'*

(Anna Miguel, teacher from Malta)

*'I think the biggest problem is the time, and time for planning IBL. Especially with key stage four (ages 14-16), you're looking at a very heavy amount of syllabus content. Fitting in time for IBL is quite hard. There's so much to cover in a very short space of time.'*

*(Brad Holmes, teacher from the UK)*

Some teachers claim that they have no time for IBL, because their syllabus is already too crowded with content. This reveals a common misconception – IBL is not additional content, but rather a different perspective on learning that provides students with a deeper understanding and appreciation of the content. In the short term, progress might seem slow. However, when technique is underpinned by deep understanding, it is retained for much longer and re-teaching becomes unnecessary. In addition, as students become more independent in their learning, they begin to learn new material for themselves and to help one another. In the long run, IBL approaches have considerable benefits – but this needs persistence.

*'Of course it takes a lot of time but it is not something additional. Actually, I learned to use IBL to work on mathematical content. Students learn things in a much deeper way and understand more.'*

(Carmen Garcia, teacher from Spain)

## 'IBL lessons are demanding on teachers' time.'

When IBL is new, teachers find preparation time consuming. This does, however, improve with experience, and the outcomes are more than worthwhile. It seems important that we take our time:

*'This is difficult at first, but you can get used to managing it, and then realise that in fact, it gets much easier.'*

*(Hajnalka Csapó, teacher from Romania)*

*'Our concern was: Is this going to be something over and above the work that we already have? If so, then it is already difficult to manage to cover everything; but then we realised that this was a normal lesson and then we were not so worried or afraid. I still would say that lessons with IBL are more time consuming. That's definitely so, but I think that at the end, they are worth it because what the student has acquired is a lot better than in a traditional lesson.'*

(Anna Miguel, teacher from Malta)

## 'IBL won't get my students through their exams.'

A main priority for teachers is in helping students to do well in their external assessments:

*'My primary task is to prepare students for the next external assessment, which gives them a certificate that helps them in their future. They don't want more - and if I did more, well the first thing they would do is rebel. The next step would be that the parents would tell me that it is not my task to do this. That's the point at which I realise I don't want to come up against a brick wall.'*

(Marianne Berger, teacher from Germany)

*'I don't have the time to do modelling and IBL. I need to prepare the students for the external assessment.'*

(Mariah Bonello, teacher from Malta)



Inspiring experiences – and challenges

It is true that in many countries, examinations do not directly reward students for their ability to inquire and problem-solve. This is an issue that some governments are aware of and are trying to address. In the current UK proposals for reform, for example, there is a new emphasis on tackling unstructured problems.

When teachers have tried to integrate IBL in a sustained way, however, they have found that exam results do improve:

*'Some of my students did not do well in their half yearly exams – they started to lose heart. However their self-confidence improved once I used IBL activities with them. Their marks on tests also showed plenty of improvement.'*

(Gabriel Abela, teacher from Malta)

As well as external assessments, many school-based tests do not reward the skills developed by IBL. This is an area that continues to challenge teachers:

*'I haven't included an extremely open task in a test. I'm not there yet. Normally, I use it as an oral assessment. I plan to try integrating open tasks in a test and look for tasks that work well. There are not enough materials for IBL assessments, but the PRIMAS Homepage provides interesting ideas.'*

(Irina Weinrich, teacher from Germany)

## 'Parents and others will not understand IBL.'

When implementing IBL, there is an understandable fear that parents and others will misinterpret our intentions and believe that we are abdicating from our responsibility to tell students how to perform scientific techniques. In our experience, however, this challenge has not been problematic when IBL has been carefully explained at parents' evenings and other events. They are reassured to find that students are still being taught the techniques!

*'I don't have any problems with the parents. I explain that I don't do IBL all the time. I mean you have to talk about it a little at a parents' evening that the teaching of maths becomes a little bit more open, but the students will still be able to calculate at the end of a term.'*

(Nina Meyer, teacher from Germany)

As students begin to enthusiastically share classroom experiences and learning with parents, parents have been reassured:

*'The feedback of parents is always positive because students enjoy the nontraditional activities and they share their experiences with their parents. I think we should use a comprehensive way of informing the parents, as well as involving them in our activities.'*

(Vadim Mazilescu, teacher from Romania)

## 'My students will not like IBL.'

IBL makes substantial demands on students as well, and they need time to adjust:

*'Students are more acquainted with traditional, transmissive lessons, so they have to learn to work in an IBL-oriented way.'*

(Martin Egi, teacher from Switzerland)

Some students, particularly those that have succeeded under traditional, transmission–oriented approaches, may take time to get used to the new, more active roles that are required of them: 'Why don't you just tell us how to do this task?' 'Why do we have to plan this study ourselves?' Such reactions are normal.
It is important to explain to students the new expectations that we have of them: that they should learn to actively ask questions; seek answers; compare approaches; and pursue their own lines of inquiry - without continually asking for help. They should also know how important it is to learn to work collaboratively, just as professional scientists and mathematicians do in the world around them.





Such changes happen only gradually, step-by-step. To start with, we may use shorter tasks that are just a little more open and unstructured than usual.

*'I tried to focus on small investigation units that can be done within one or two lessons. They have the advantages that students can follow the activity and can see the results.'*

(Pavol Jaroslav, teacher from Slovakia)

Students may be asked to provide more reasoned explanations for their approaches and results. When they are stuck, then the teacher doesn't help too quickly, but allows time for thinking before offering help.

*'I ask students to exemplify or to detail their arguments. In most of these cases they are able to proceed further. If the student can't go on, I try to change the context in order to emphasise the problem. These modified contexts usually are related to everyday life.'*

(Cosmin Mihai, teacher from Romania)

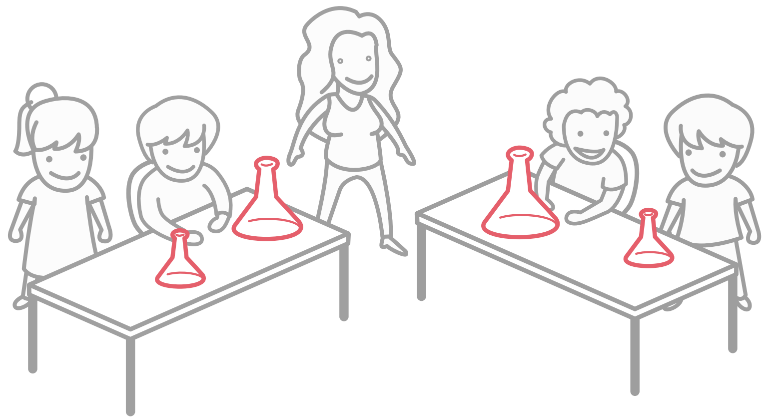
Questioning progresses during an inquiry, and we have found it helpful to plan these carefully in advance. Typically, at the beginning of an inquiry, we ask questions like: 'How can you simplify this problem? What assumptions might be made?' Then, after the student has formulated the problem, we may continue: 'Can you think of a systematic approach? What is a sensible way to record your data?' As data is collected, we ask: 'Can you see any patterns here? Can you explain why these arise?' Towards the end, our focus is on communicating the findings: 'How can you explain this clearly and succinctly?'

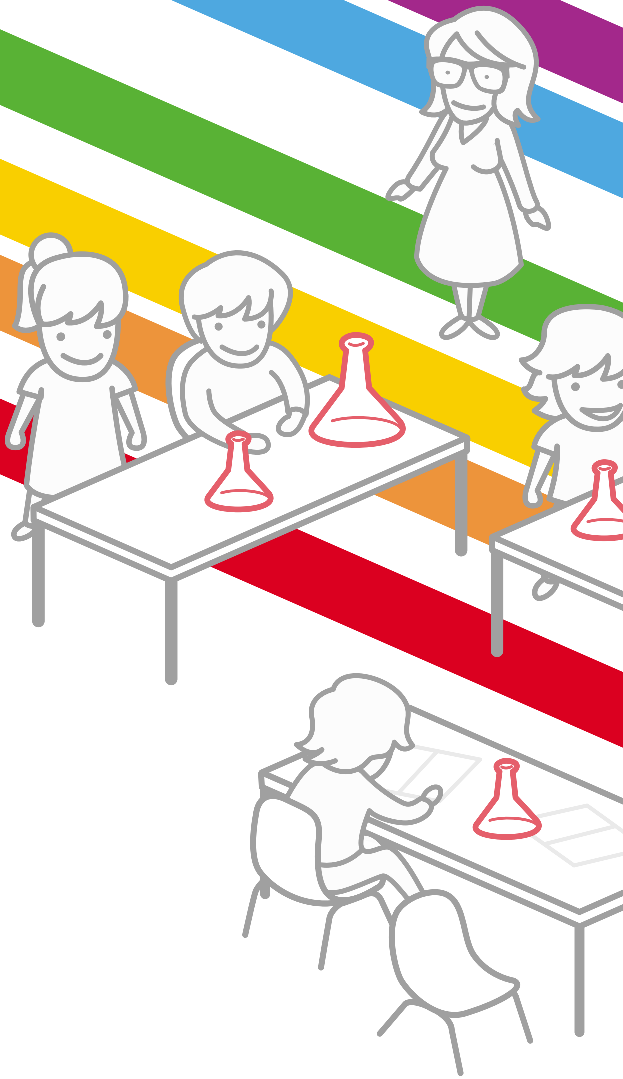
Ultimately, students do come to appreciate why we are teaching them through this approach:

> *'I think that IBL is the way forward for teaching mathematics and science. It enables students to become critical and creative thinkers capable of using mathematical and scientific skills to solve problems. Moreover, it helps students to gain the ability to communicate to others mathematically or scientifically without fear and with a lot of confidence.'*
>
> (Gabriel Abela, teacher from Malta)

The attainment of such desirable outcomes though the wide-spread implementation of inquiry-based learning in our mathematics and science classrooms certainly involves challenges. However, when we consider the competences students acquire when learning in an IBL way, the benefits to them – and to ourselves as teachers - far outweigh the task of facing and overcoming obstacles and (initial) resistance. Using small steps to try and implement IBL is a method that works well as a starting point. The experiences of those involved in the PRIMAS project clearly show that the results of doing so are well worth the effort involved.

In conclusion, we would like to take this opportunity to thank the teacher educators, the pre and in-service teachers and their students who participated in PRIMAS. Their contributions enabled us to gain fruitful insights into their work practices – and were the basis of the project's success. Together, we will continue to promote inquiry in mathematics and science education across Europe – and in doing so, welcome your own feedback and involvement to help further our goal of preparing our students for their lives beyond our classrooms.





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